MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

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MEETING OF 1935 JUNE 14

Mr. J. H. Reynolds, President, in the Chair.

The election by the Council of the following Fellow was duly confirmed :—

Reginald Douglas Kreltszheim, B.Sc., F.R.Met.Soc., Junior Technical Assistant, Colombo Observatory, Ceylon, and 72 Claremont Road, Highgate, London, N. 6.

One hundred and five presents were announced as having been received since the last meeting, including :—

Yale University Observatory, General Catalogue of Stellar Parallaxes, by Frank Schlesinger, with the collaboration of Louise F. Jenkins. Second edition, containing all determinations available in January 1935.

The President referred to the recent deaths of Dr. E. B. Frost and Dr. C. St. John, Associates of the Society.

Dr. Henry Norris Russell delivered the George Darwin Lecture, taking as his subject "The Analysis of Spectra and its Applications in Astronomy."

Dr. W. S. Adams (Director of the Mount Wilson Observatory) gave a brief account of the progress of the construction of the 200-inch telescope. Dr. S. A. Mitchell (Director of the Leander McCormick Observatory) and Dr. J. Jackson (H.M. Astronomer at the Cape) spoke on the work of their respective observatories ; and Professor B. Boss (Director of the Dudley Observatory) announced the forthcoming publication of his new General Catalogue of Stars.

THE ANALYSIS OF SPECTRA AND ITS APPLICATIONS IN ASTRONOMY.

(*George Darwin Lecture, delivered by Dr. H. N. Russell, Assoc.R.A.S., on* 1935 *June* 14.)

I. *The Analysis of Spectra*

Principles.—Spectrum analysis, as the words were understood a generation ago, was a purely empirical science. We could observe the lines of the various elements, accurately for position, and roughly for intensity, but our results were almost purely taxonomic. We knew that sodium had few lines in its spectrum, calcium more, and iron a host; but we had no more idea why this should be so than why men should have two legs, spiders eight, and centipedes a great many. Yet the achievements of this elementary knowledge were permanent and great, culminating in Rowland's identification of nearly twoscore elements in the Sun, the recognition of helium there long before it was "run to earth," Lockyer's discovery of the dissociation of atoms at high temperatures in the stars and under high excitation in the laboratory; and, above all, the most impressive of all proofs of the essential uniformity of nature, to the utmost limits of telescopic observation. These results are all the more remarkable because they were attained without any real understanding of the nature of the process by which spectral lines were emitted or absorbed, or of the relation between the various lines of a given spectrum.

The beginning of the other type of analysis of spectra, which occupies itself with these problems, may well be set in 1883. It was already well known that the spectra of certain elements, such as sodium and magnesium, showed conspicuous pairs or triplets of strong lines. The separations of the various groups, when measured in wave-length units, are quite different, but Hartley, in that year, noticed that, when the number of waves per centimetre (wave-number) was used instead of the number of centimetres per wave, the distinctive groups for each element had identical separations, showing that some real relation existed between them. Two years later Balmer came upon his famous formula, which represented with remarkable precision the wave-lengths of the hydrogen lines known in the laboratory, and of the long extension of the series in the ultra-violet, found in the first-type stars.

A few years later Kayser and Runge recognized many other series in the spectra of the alkali and alkaline-earth metals, some composed of pairs, some of triplets, but all alike with lines growing more closely spaced and fainter for shorter wave-lengths. These series were at first expressed by apparently unrelated empirical formulæ, but in 1890 Rydberg found that they could usually be closely represented by the general formula

$$n = n_\infty - \frac{R}{(m+\mu)^2},$$

where n is the wave-number, n_∞ and μ constants peculiar to the series, m an arbitrary integer, and R a universal constant, common to all spectra. Here at last was evidence of an underlying unity beneath the superficial diversity.

The independent discovery by Rydberg and Schuster, in 1896, that the constant part n_∞ of the formula for one series was identical with the variable part in the expression for a particular line of another series, led to the combination of the various series formulæ into the equation

$$n = \frac{R}{(m_1+\mu_1)^2} - \frac{R}{(m_2+\mu_2)^2},$$

where for a particular series m_1 is constant and m_2 has successive integral values.

Finally, Ritz in 1908 recognized that by giving increasing integral values to both m_1 and m_2 many unclassified lines (especially in the infra-red) could be brought into the scheme. His combination principle may be stated. The wave-number of any spectral line may be exactly expressed as the difference of two *spectroscopic terms*, which are constants characteristic of the atom—and, as we now know, of molecules as well.

Rydberg's expression $\frac{R}{(m+\mu)^2}$ represents a series of such terms, possessing similar properties. Precise measurements show that μ is never exactly constant (except for hydrogen when $\mu = 0$), though it usually varies but little.

The most satisfactory general formula is that of Ritz, which may be written $\mu = \mu_0 + \alpha T$, where T is the term value itself, or $T = \frac{R}{(m+\mu_0+\alpha T)^2}$, μ_0 and α being constants. The direct solution of this equation is unhandy, but is now quite unnecessary since tables are available † giving the values of R/n^{*2} for intervals of 0·001 in the denominator n^*. Having the wave-numbers of the lines of a series, we start with an approximate value of n_∞ (which is easily obtained) and thence derive T and n^*. According to Ritz's formula, n^* should be a linear function of T (or of $n = n_\infty - T$). Plotting, with a trial-and-error adjustment of n_∞, gives a far quicker solution than any algebraic process, and has the great advantage of taking into consideration *all* the lines of the series, and even rival candidates for a given place. Since the test of the formula is the collinearity of the plotted points, three lines of the series are necessary to establish the constants, and a larger number is desirable. There are a good many series, definitely shown to be such by other evidence, which fail to satisfy the Ritz formula. They can often be represented pretty well by a formula due to Langer ‡

$$n^* = m + \mu_0 + \alpha T + \beta/(T - T_0),$$

where T_0 is the term value for some perturbing term of similar nature, but

† Princeton University Observatory, 1934.

‡ *Phys. Rev.*, **35**, 649, 1930.

not belonging to this series. When two or more perturbing terms are present, exact calculation is hardly practicable.

After this digression—which has led us far ahead in the historical order —we may return to the greatest of all advances in atomic theory, Bohr's fundamental postulate (1914), which in somewhat modernized statement is: Every spectral line is emitted (or absorbed) in connection with a transition in the atom (or molecule) which produces it between two definite energy states, and the frequency ν of the radiation is given by the equation $h\nu = E_1 - E_2$ where h is Planck's constant, and E_1 and E_2 the energies of the atom in its initial and final states. If c is the velocity of light, the wave-number $n = \nu/c$, we have $n = \frac{E_1 - E_2}{ch}$. Spectroscopic terms are therefore characteristic of definite states of the atom, and afford a very exact measurement of their energy-content.

This portion of Bohr's theory has withstood the test of twenty years' further investigation, and appears to be fully established. The other portion, which associates these energy-states with definite orbits of the electrons about the nucleus, has been superseded by wave-mechanical considerations, more in accordance with the facts, but far less easy to picture. As generalized by Sommerfeld, it still remains much the best device for interpreting and remembering the somewhat intricate relations between atomic and spectral structure, and it will therefore be adopted here without further apology.

In an atom such as sodium, the electrostatic field due to the charges upon the nucleus and all but the outermost of the electrons is replaced for simplicity by a radially symmetrical field. The terms of each spectral series are then found to correspond to orbits of the outer electron, all having the same (quantized) angular orbital momentum but with the radial momentum, and the maximum distance, increasing with the serial number m. To the series limit correspond the recession of the electron to infinity—that is, the ionization of the atom. A series term, therefore, represents the energy necessary to ionize the atom, provided it is already in the given state. It is measured *downward* from a state of high energy-content.

For many purposes (to diverge for a moment) it is more convenient to use the energy-content measured *upward* from the ground-state of the atom, in which it possesses the least possible energy consistent with its maintenance of its degree of ionization. In almost all well-investigated spectra, the term value for the ground state and the others can be connected by a chain of combinations involving observed lines, so that their relative values are fixed with the extreme precision of modern spectroscopic measures. To find the ionization energy demands recourse to a series limit, which must be determined by a process of extrapolation. In a few spectra where long and regular series are known (57 consecutive terms for sodium, 41 for lithium) the limit may be very precisely fixed, but when only a few members of the series are known its determination is much less accurate. This is especially true when the series are perturbed, as happens in most of the complex spectra.

Energy-levels and spectroscopic terms measured up from the bottom

may be evaluated once for all (barring changes in the last decimal from more accurate measures). Those referred to series limits are subject to numerical change whenever new members of the series are discovered. The former are therefore much better adapted for tabular purposes, except for a few spectra.

Bohr's theory went further, predicting that a singly ionized atom, in which but one "outer" electron remains, should have a spectrum of exactly similar structure, but with the constant in the series formulæ $4R$. This prediction was immediately verified by A. Fowler,* who found a beautifully complete set of series in the spark spectrum of magnesium, and others, less extensive, for the other alkaline earths. The lines discovered in 1896 by Pickering in stars of Class O, and reproduced in the laboratory by Fowler in 1912, were by Bohr's theory conclusively identified as due to ionized helium. A minute difference in the Rydberg constant, for this and for hydrogen, was triumphantly explained by the inclusion in the equations of the motion of the atomic nucleus. Series with constants $9N$, $16N$, from doubly and trebly ionized atoms, were detected by Fowler in 1924.

The "astronomical atom" now accounted for all the features of these spectra except the duplicity of most of the terms. This was accomplished by Uhlenbeck and Goudsmit in 1925, by assuming that the electron itself was spinning. Forward and backward rotation accounted for the separation of the pairs, quantitatively, and for the apparent exception that the terms of one series (*S*-terms) were not double. This made the astronomical model look better than ever; but when it came to two-electron systems, such as neutral calcium, things were not so satisfactory. There are here two whole systems of series (as Saunders pointed out in 1910), one of single terms, the other of terms usually triple, with weak intercombinations between the two. To account for these it was necessary to assume that the axes of spin of the two orbital electrons were set either in just the same or exactly the opposite direction. In the first case, Landé's vector-model requires that the inclination of their common equator to the orbit-planes shall be either 0°, 120° or 180°, giving three neighbouring energy-values and triple terms —in the second, the terms are single. Moreover, the spectra of calcium, strontium and barium contain many strong lines which do not fit into the regular series at all. Russell and Saunders (1925) associated these with states in which both the outer electrons are excited—that is, in larger orbits than normal. The mutual inclination of these orbits is then quantized, giving rise to triads of closely related terms. Some of these terms are triple, which was later explained by quantization of the inclination of the coincident equatorial planes of the two electrons to the invariable plane intermediate between those of the orbits.

This model, while still mechanical, can hardly be called astronomical. Though now superseded, it led to the recognition that the "anomalous" terms themselves belonged to series obtained by leaving one of the two electrons in its original excited orbit, and moving the other out through a series of such orbits. At the limit this leads to an atom which is ionized

* *Phil. Trans.*, A, **214**, 225, 1914.

and also excited ; this limit, indeed, is not the ground-term of the ion, but its next highest energy-state. On the way to this, the atom, while still neutral, passes through a series of states in which it contains more than enough energy to ionize it in the orthodox fashion. The first series of this sort was detected by Russell and Saunders in calcium. Many others are now known. One in *Cu* I has five known members closely following a Ritz formula, but all above the level of ordinary ionization.

The extension of these principles to the more complex spectra was foreshadowed as long ago as 1893 by Kayser and Runge's discovery that the spectra of several elements, such as tin, lead and bismuth, though showing no recognizable series, contain groups of lines whose wave-numbers differ by exactly the same amount. Such lines represent transitions between two low levels separated by the constant difference, and a number of higher ones. This could only be realized after Bohr's work ; but, in the meantime, a great number of constant differences were recognized in many spectra by Rydberg, by Paulson, and especially by Snyder, who developed remarkable arrays of connected differences in the spectra of rhodium and of some other elements, which really went far towards an analysis of the spectral terms.

More general relations were recognized in the alternation and displacement laws of Kossel and Sommerfeld (1919). The first of these states that terms of odd and even multiplicities are found successively when the elements are arranged in order of atomic number, and the second, that the spectrum of any ionized atom resembles in general structure that of the neutral atom of next preceding atomic number. The latter is a natural consequence of Bohr's postulates, since the two have the same number of electrons outside the nucleus, and may be expected to have fairly similar structure.

The clue to the maze was found at last in 1922 by Catalàn, who recognized, in the arc spectrum of manganese, groups of intimately related lines, which he called multiplets, and which prove to be the structural units of the more complex spectra.

The work of Sommerfeld and Landé soon provided an interpretation of these groups in terms of atomic structure. The orbital angular momenta of the various active electrons combine into a resultant L which may have the values 0, 1, 2 . . . in terms of the natural unit $h/2\pi$. The spin momenta combine to a vector S, which may have the value 0, 1, 2 . . . or $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$ . . . for even or odd numbers of electrons. The vectors L and S then combine to give a resultant J which is the angular momentum of the atom. Changes in the angle between L and S give several permissible values of the inner quantum number J, belonging to the components of a multiple term. Transitions between two such terms give a multiplet of spectral lines. The value of S determines the multiplicity of the term (singlet, doublet, triplet), that of L its type (the S, P, D, F terms of the ordinary notation). Both these are constant for all members of a series.

Though the model of spinning electrons moving from place to place along definite orbits has passed, the angular-momentum vectors l and s

for each electron, and their resultants L and S, appear to be real. Increasing values of the total quantum number n, which differs by an integer from m in Rydberg's formula, are now associated not with orbits extending to greater distances, but with a probability distribution of charge extending farther from the nucleus.

In the heavier atoms the coupling of the vectors into resultants is more complicated, but the results are not essentially different. If, in atoms containing many electrons, the combination of the vectors was unlimited, the number of energy-states, and still more of spectral lines, would be enormous, and the spectra would be far more complicated than they are. The intelligibility of actual spectra depends on the existence of two sets of restrictions: one limiting the number of states, the other that of transitions between them.

The first of these, Pauli's restriction, is of fundamental importance. No two electrons in the atom may have "exactly similar orbits"—that is, no two can have identical values of the numbers n, l, s, and of a fourth quantum number m defining the projection of l upon the axis of an external magnetic field. This principle determines the existence and numbers of the successive shells of electrons, which had already been postulated empirically by Bohr to explain the periodic system of the elements. The resultant vectors for the electrons of a closed shell are zero, hence the inner shells in an atom take no part in determining the number and nature of the spectroscopic terms, though they have an influence upon their absolute values. Atoms whose outer electrons are similar in number and l-values have similar spectra, as well as similar chemical properties.

When two or more similar electrons (having the same values of n and l) are present outside the closed shell, Pauli's principle restricts severely the possible number of resulting pairs of values of L and S (that is of spectroscopic terms). A shell complete except for one electron gives but one doublet term, as does an isolated electron. The maximum complexity occurs when a shell is half or a little more than half complete. The greater the l-values of the active electrons, the more complex the spectrum. Thus oxygen with four p-electrons ($l=2$) has a fairly complex spectrum, but is far exceeded by iron with six d-electrons ($l=3$), while the rare earths, which contain an incomplete group of f-electrons ($l=4$), have much the most intricate of all spectra.

The consequences of Pauli's principle have been worked out in full detail by Hund (1927). The resulting theory is one of the most satisfactory in the whole realm of physics. Applied to one after another of the more complex spectra, it has accounted with complete success for all the terms that have been observed, and led to the discovery of new ones. Only a single apparent exception—an unexplained energy-level in *Pd*I*—has been found among the thousands of such levels which have been fully interpreted.

Despite these limitations, the number of lines in the more complex spectra would still be intractably great, were it not for other restrictions, which greatly reduce the number of observable combinations (*i.e.* transitions

* Shenstone, *Phys. Rev.*, **36**, 674, 1930.

which lead to radiation of perceptible strength). The most important of these are as follows. Combinations normally occur :

I. When the sum of the l-values of the individual electrons is even for one state and odd for the other.
II. When the values of $\mathcal{J}$ differ by 0 or 1.
III. When the values of S are the same.
IV. When the values of L differ by 0 or 1.

The first two of these rules are very rarely transgressed. The "forbidden lines" which violate I appear only in sources of very low density, such as the nebulæ. Rule III is rarely violated for small atomic numbers, and the resulting intersystem lines are faint. For heavy atoms it is hardly effective at all. About the same is true of Rule IV. The spectra of these elements are considerably more difficult to analyze than those of lighter elements of similar structure. The heaviest of all, *Th* and *U*, have very complex spectra, which have not yet been disentangled, even in part.

The spectra of molecules present still greater complications. In addition to the electron "jumps" which occur in atoms, molecules may alter the state of vibration of the various nuclei about their equilibrium positions, or of rotation of the system as a whole—both of which are quantized. Even for diatomic molecules, a great variety of intricate possibilities occur, which are now well understood theoretically. Changes in rotation alone give the individual lines of a band ; those in vibration, different bands of a system ; while each electronic transition gives a whole system of bands, usually in different parts of the spectrum.

In a small but increasing number of cases, such as H_2, He_2, C_2H_2, series of electronic transitions, represented by formulæ of the Rydberg type, have been discovered. The limit of the theoretically infinite series of configurations is the ionization of the molecule without dissociation. A considerable number of band spectra have been definitely attributed to ionized molecules, such as N_2^+, CO^+.

Molecular spectra also exhibit important series of transitions of another sort, in which the upper or lower vibrational state changes. Such series have only a finite number of members, and end when the oscillations become so great as to separate the two atoms, dissociating the molecule without ionizing it. This affords a powerful spectroscopic method for determining the energy of dissociation. Unfortunately, extrapolation from the first few terms of such a series to the limit is less reliable than for the majority of series in atomic spectra, and accurate values of the dissociation energy have so far been derived in only a few cases, where long series of absorption bands, arising from transitions up from the ground-state of the molecule, have been observed. One, or perhaps both, of the atoms resulting from the dissociation may be liberated in an excited state, which must be taken into account in discussing the energy relations.

Molecules, like atoms, may be charged with more energy than is necessary for the easiest form of dissociation, and yet remain bound together long enough to produce spectral lines. In such cases the mean life is usually

short, and the lines are diffuse. This "predissociation" is of much aid in estimating dissociation energies.

For polyatomic molecules the complexity is still greater. Several independent fundamental modes of vibration may exist, and their higher harmonics and combinations give rise to a great number of band-systems. The rotation of the unsymmetrical body produces a very complicated situation, and the resulting relations of individual lines in the bands have only begun to be interpreted. Good progress has already been made in the interpretation of the spectra of important molecules like H_2O, NH_3 and CH_4.

It should also be mentioned that both atoms and molecules may emit or absorb spectra, which are truly continuous, but diminish in intensity so much, outside a given range, that they are practically confined to specific regions of the spectrum. In atoms these spectra are found beyond series limits (on the short-wave side), and correspond (on the orbital picture, and in absorption) to the transition of an orbit from a definite elliptic orbit within the atom to a hyperbolic orbit, passing off to infinity with an amount of kinetic energy which may have any value within wide limits—in other words, to photo-electric ionization. Absorption of this sort is now believed to play the principal rôle in the general opacity of stellar atmospheres. The reverse process of capture gives rise to continuous emission which has been observed in gaseous nebulæ.

In molecules, continuous spectra often arise from transitions to unstable states, in which the nuclei repel one another instead of attracting. Such transitions produce continua, often of considerable intensity and rather short range, which are prominent in the laboratory, but have not yet been detected in celestial spectra.

In conclusion, mention should be made of X-ray spectra, which give information regarding the binding energies of the inner electrons, belonging to completed shells, just as the optical spectra do for the outer electrons. These have no direct astronomical applications, since, fortunately for us, no short-wave radiation penetrates the Earth's atmosphere. But the possibility of calculating how completely an atomic nucleus will be stripped of its more firmly bound electrons at very high temperatures and pressures has been of fundamental importance in the theory of the internal constitution of the stars.

Practice.—In 1922, when Fowler published his classic monograph on "Series in Line Spectra"—just before Catalàn's discovery of multiplets—fairly adequate, though not always complete, analyses were available for the arc spectra of about a score of elements and the first spark spectra of half a dozen. At the present time comparable analyses have been made for between sixty and seventy neutral atoms, and for fifty in the singly ionized state, and also for about sixty spectra corresponding to higher degrees of ionization. Most of the observational data required for these analyses were available thirteen years ago. When the principles and methods of analysis were understood, the details came with a rush.

(1) The prime requisite for the analysis of a spectrum is of course an accurate and complete list of wave-lengths. The necessary degree of precision varies with the number of lines. When these are few, the chances of being misled by accidental numerical coincidences are small; in very rich spectra (as for the rare earths) these chance agreements will be very confusing unless the wave-lengths are good to a few thousandths of an angstrom. It is even more important that the list shall be complete, for faint lines often provide the test whether a given arrangement of the stronger ones is correct. For this very reason, one can hardly hope for complete success in classifying all the fainter lines of really complex spectra, for many of these cannot be put in their places without the aid of still fainter ones, and the long exposures required to detect these bring out a host of others.

It is not at all unusual to hit upon some tentative new term which is satisfactory, except that it predicts one or two lines which should be of fair strength, but are not in the list, and then to find that these lines are either (*a*) so close to a stronger one of the same spectrum as to be masked, or (*b*) similarly masked by a line of the spark spectrum of the same element (if one is working on the arc, or *vice versa*), or (*c*) practically coincident with a line of some other element, and so rejected from the list as due to impurity. It therefore pays the investigator to be tolerant in making up his working list, and include even the "impurity lines," with notes of their suspicious origin.

For this reason the old lists of Exner and Haschek are still of great value. They are not of high precision—though remarkably good for measures made upon the image of the original negative projected with a steropticon—but they often contain many real lines which the later investigators have missed.

A wide range of wave-length is nearly always necessary for a complete analysis. The new infra-red sensitive plates have been of great service in this connection, and every new advance is eagerly welcomed. Observations in the far ultra-violet with vacuum spectrographs are essential to the full understanding of the spectra of some neutral atoms, such as all the permanent gases, many of those which are singly ionized, and all those in higher stages.

The task of turning wave-lengths into wave-numbers, once very laborious, has been greatly simplified by Kayser's tables with Babcock's ingenious adaptation for their use in the infra-red, beyond λ 10,000.

(2) Second in importance are the *intensities* of the lines, which afford the principal criteria for the arrangement of the lines into multiplets. The qualitative empirical rule that the lines of the principal diagonal of a multiplet are strong, the first satellites much weaker, and the second satellites very faint, has been replaced by the detailed numerical formulæ which were developed simultaneously and independently by Kronig, Sommerfeld and Hönl, and Russell. Though these formulæ are not rigorously exact—or even approximately so for large atomic numbers—they are usually important guides.

It is remarkable that, until very recently, spectroscopists were content with so crude a specification of line-intensities. The assigned "intensities" were rough estimates, based on the appearance of the lines on the negatives.

The scales used by different observers differed radically ; and even those of the same observer were far from homogeneous for different parts of the spectrum. The older scales, too, were far too compressed, ranging only from 0 or 1 for the weakest lines to 10 for the strongest, though observers nowadays go up to 100 or even to 1000.

The analyst of the spectrum must become familiar with the significance of the scales of the various observers (though a formal attempt to reduce them to a homogeneous scale would hardly pay), and must also take into account the steady falling off of the recorded numbers as the limit of observation in the infra-red or ultra-violet is approached. Within the last few years reliable photographic methods of intensity-measurement have been developed, notably by Dorgelo and his associates in Holland, and applied with various refinements by Harrison and others. The chief difficulty—the self-reversal of the stronger lines—is hard to overcome ; but there is good reason to hope that within a few years we may have real and trustworthy measures of the intensities of the lines in the more important spectra, and especially in that of iron—where the existing theoretical formulæ often fail, owing to large perturbations.

(3) The relative intensities of lines, both in laboratory and astronomical spectra, however, vary widely with the conditions of excitation. The third main prerequisite for the analysis of a spectrum depends on this fact, and may be described as the *temperature classification* of the lines. The phrase is derived from the pioneer work of A. S. King, who observed that, when the spectrum of an element was produced in the arc, and in the electric furnace at various temperatures, the relative intensities of the lines varied greatly, some fading out rapidly as the temperature fell, and others (not always the strongest arc lines) being very persistent. Similar phenomena had been observed long before by Lockyer and others, but King was the first to make a systematic study. Before the days of modern spectroscopic theory his classification provided the first conclusive proof that sunspots were cooler than the rest of the disk. It gained in importance when, in analysed spectra, it was found that the low-temperature lines of classes I and II had, as the lower level of transition, one of the lowest energy-levels of the atom, while for classes III, IV and V, this level, on the average lay increasingly higher.

The analyst of the spectrum, therefore, may start with the low-temperature lines alone. Recurring constant differences among these will give the relative positions of low energy-levels, and, with this start, rapid progress can usually be made.

Thirty years of King's assiduous work have resulted in temperature classification, on a uniform basis, for a majority of all the elements, including some of the rarest, such as europium and rhenium. His results have been of inestimable value in the analysis of complex spectra, and certainly contributed more than any other one observer's work to its success.

Other physical properties of the lines are of value in the analysis. Self-reversal is good evidence that the lower level in the line transition is near the bottom. Diffuseness indicates that the upper level is high, and that it belongs to an atomic configuration containing an electron with a large orbit

(to use the old picture). Diffuseness, and the more unusual asymmetry, are properties common to all the lines of a multiplet, and are often of great aid in sorting them out. The most important of all the physical properties, and the earliest to be recognized, is of course the enhancement in the spark, characteristic of lines of the ionized atom. There is always some, and usually a good deal of ionization in the arc, so that many enhanced lines appear in arc spectra, and have to be weeded out by comparison with the spark, if the spectrum of the neutral atom is being analyzed.

An equivalent of "temperature classification" can be made for enhanced lines by observing their relative intensities in the arc and spark. Lines arising (if for the moment we consider the upward, or absorption, transition) from the lowest levels of the ion are strong in the arc, and often appear in the furnace (sometimes at low temperatures, like the *H* and *K* lines of Ca^+). They are frequently reversed in the spark. Lines arising from higher levels in the ion are absent from the furnace, faint and sometimes absent in the arc, and unreversed, and often hazy, in the spark. Several intermediate degrees of behaviour can often be recognized.

For the permanent gases, the arc and furnace are hardly available as sources; but different degrees of excitation can be obtained by varying the discharge conditions in a vacuum tube. Self-induction diminishes the disruptiveness of the spark, and lowers the ionization. This method is applicable even to vacuum sparks, and is a valuable aid in disentangling such spectra as that of the sodium vacuum spark, where five or six different stages of ionization may simultaneously be present.

(4) Last among the major aids in the analysis is the Zeeman effect. When the light source is situated in a strong magnetic field, each energy-level breaks up into a group of equally spaced components, and each line is converted into a very narrow multiplet. When this is resolvable, the number of components, and their separations (Landé's *g*-factor), can be determined for both the levels involved, and these usually suffice to determine $\mathfrak{J}$, *L* and *S*, for each and thus identify their nature completely. Unresolved patterns (which are in the majority, even with strongest attainable fields) can be used to determine the *g*-value for one level when that for the other is known,* and the magnetic analysis may thus be extended to all the important levels.

These data are of great aid in grouping the numerous levels in a complex spectrum into multiple terms. In the earlier analysis they often tell the investigator where to start, and so lead to the discovery of new levels. When the multiplet intervals are small, so that many individual groups stand out clearly, an analysis can be made without this aid; but when the intervals are wide, and the various multiplets intermingled, the Zeeman effect is of primary importance. Landé's formula for the *g*-factors, upon which the foregoing is based, fails in certain cases, where two neighbouring levels share their *g*'s and most of their other properties. They suffice, however, to determine the nature of the lower energy-levels for even the heaviest atoms. Among the higher levels in these cases, the perturbations are large and the whole scheme of classification of lines into multiplets, and of levels into

* Shenstone and Blair, *Phil. Mag.*, **8**, 765–771, 1929.

terms named in the conventional fashion, almost breaks down; but the identification of the energy-levels themselves remains unimpaired.

(5) Hund's theory should finally be mentioned as a main guide, telling the analyst what terms to expect, and how many, and of what nature, remain to be found. Rules defining the relative levels of the terms resulting from the same electron configuration were found empirically by Hund, and have been given a partly quantitative form by several later investigators. Many valuable hints may also be obtained by comparison with other related spectra. The comparison may either be made vertically, with elements in the same column of the periodic table, or horizontally, with elements in the same row. In the first case, the resemblance of different spectra, such as *Ca*I, *Sr*I, *Ba*I and *Ra*I, is almost uncanny, extending to the finer details; but in the heavier elements the multiplet separations are much wider, and homologous lines lie at longer wave-lengths. In comparing the spectra of elements of successive atomic number, such as those from *Ca* to *Zn*, the corresponding groups are those which arise from the same electron-jump. The number, series type, and multiplicity of the terms involved differ according to Hund's rules, but the lines lie in the same part of the spectrum (often shifting gradually toward the violet with increasing atomic number) and are characterized by similar properties, such as sharpness or diffuseness.

In the study of highly ionized atoms, still another principle is employed. In isoelectronic sequences, *e.g.* *Na*I, *Mg*II, *Al*III, *Si*IV, etc., the wave-numbers of homologous lines often form series with small and nearly constant second differences (first noticed as the "irregular" doublet-law for X-ray spectra). The position of anticipated multiplets in spectra still higher in the sequence can thus be predicted within a few angstroms, so that they can be picked out in the hot spark from the confused mass of superposed spectra belonging to several different degrees of ionization.

When all is said and done, the analysis of a complex spectrum is curiously like the solution of a glorified cross-word puzzle. There are usually a few places where a start is easy. Once a word or two has been written in (or a couple of low-energy levels found) many other words (or spectroscopic terms) can be fitted in, successively, each affording a clue to others. One must of course know the pattern of the puzzle (the Hund theory). The various spectroscopic data (temperature class and Zeeman effect) may be compared with the definitions of the words of the puzzle (though they are often more similar to the picturesque hints of an English puzzle than to the elementary directness of many in American newspapers). There are even unkeyed letters in the puzzle, or lines in the spectrum, which belong to one word only (or to a level having no other combinations) and must be identified by "making sense." The main difference is that the spectral puzzle takes longer to solve, and that, when you are through with it, you can publish the result and call it research!

Results.—Progress in this work is still so rapid that a summary of its situation will be out of date almost before it can be printed. Nevertheless, it may be permissible to offer a survey of the status of analyses of atomic

spectra in the spring of 1935. Borrowing the schoolmasters' scheme of grading, a substantially satisfactory and complete analysis has been denoted by A; one in which the main features are settled, but important details remain to be cleared up, by B; while C signifies that the analysis is seriously incomplete, D that only a beginning has been made, and O that nothing seems yet to have been done. Unpublished work by many investigators has been included when its results were known.* The list makes no claim to be exhaustive, and its compiler begs the indulgence of any whose work he may have overlooked. The assignment of grades, and especially of the intermediate symbols (which run A, A^-, B^+, B, B^-, etc.) might be made differently by another critic; but it is hoped that the general state of present knowledge is correctly represented.

Table I.

Element	I	II	III	IV	Element	I	II	III	IV
H	A	...	...	...	*V*	A	C^+	C	C
He	A	A	...	...	*Cr*	A^-	B^+	C	C
Li	A	A	A	...	*Mn*	A	C	C	D
Be	A	A	C^+	A	*Fe*	A^-	A^-	O	D^+
B	A	A	A	C^+	*Co*	A	B^-	O	O
C	A	A	A	A	*Ni*	A	A	D	O
N	A^-	A	A^-	A	*Cu*	A	A^+	C^-	O
O	A^-	A	A	A	*Zn*	A	A	C	?
F	B	A	A	A	*Ga*	A	A	C^+	D
Ne	A	A	B	C	*Ge*	A	B	C	C
Na	A^+	A^-	A^-	A	*As*	B^+	C^+	B	D
Mg	A	A	A	A^-	*Se*	A	B	C	D
Al	A	A	A	A	*Br*	A^-	C^+	C	D
Si	A	A^-	A	A	*Kr*	A	A	A	D
P	B^-	A^-	B	D^+	*Rb*	A	B	?	?
S	A	B	B^-	C	*Sr*	A	A	?	?
Cl	A	B^+	A^-	C^+	*Y*	A	A	A^-	?
A	A	B^+	B	B^-	*Zr*	A^-	A	A^-	A
K	A	B	C^-	C^+	*Cb*	D	D	D	D
Ca	A	A	B	C^-	*Mo*	A^-	D	D	C
Sc	A^-	A	A^-	O	*Ma*	...	...	...	...
Ti	A	A	A^-	A	*Ru*	B	C	O	O

* The writer is much indebted to Messrs. Meggers, Kiess, Gibbs and Dobbie for unpublished data.

Element	I	II	III	IV	Element	I	II	III	IV
Rh	B	C	O	O	*Tu*	O	O	O	O
Pd	A	A	O	O	*Yb*	B	C	O	O
Ag	A	B+	B	?	*Lu*	B	A−	B	O
Cd	A	A	C	?	*Hf*	B	A	C	O
In	A	A	B−	D	*Ta*	C	O	O	O
Sn	A−	A−	B−	D	*W*	B	O	O	O
Sb	B	B	B	C	*Re*	A−	O	O	O
Te	A	O	O	D+	*Os*	B	O	O	O
I	B−	D+	D−	O	*Ir*	C+	O	O	O
Xe	A	C	O	O	*Pt*	A	O	O	O
Cs	A	C+	D	?	*Au*	A−	C	O	O
Ba	A	A	O	D	*Hg*	A	A	D	D−
La	A−	A	A	O	*Tl*	A	B	B−	D
Ce	D	D	D	C	*Pb*	A	A	C+	B
Pr	O	O	O	O	*Bi*	A−	A	A	B
Nd	O	O	O	O	*Po*	D−	O	O	O
61	...	...	...	...	85	...	...	...	...
Sa	B−	O	O	O	*Rn*	A	O	O	O
Eu	B−	A−	O	O	87	...	...	...	...
Gd	D	O	O	O	*Ra*	A	A	O	O
Tb	O	O	O	O	*Ac*	...	...	...	...
Dy	O	D−	O	O	*Th*	O	O	O	O
Ho	O	O	O	O	*Pa*	...	...	...	...
Er	O	O	O	O	*U*	O	O	O	O

For the first fourteen elements (including silicon) all analyses have been pushed about as far as possible except for *F*I, *Ne*III and *Ne*IV; and the admirable studies of Edlèn * and Soderqvist † at Upsala have extended our knowledge far beyond the limits of the table, and even as far as *Al*x—an atom nine times ionized. The displacement law is beautifully verified, the spectra of aluminium in successive degrees of ionization resembling in structure the arc spectra of all the preceding elements as far as beryllium. Similar studies will probably soon be made for the rest of the second short period.

In the first long period—beginning with calcium and scandium—the spectra become much more complicated. The arc spectra are pretty well

* *Nova Acta Regiæ Societatis Scientiarium Upsaliensis*, 9, No. 6, 1934.

† *Ibid.*, 9, No. 7, 1934.

cleared up, though something remains to be done for chromium, iron and manganese. The spark spectra, especially that of manganese, need more attention, though one of the most important, *Fe*II, is being very successfully analysed by Dobbie. Many of the higher spectra have not been attacked. The most urgent need is for an analysis of *Fe*III. Its great complexity, and the difficulty of separating its lines from those of still higher states of ionization, have deterred vacuum-spark investigators. The analyses for the second long period lag definitely behind the first. Masurium, of course, is left blank, since no sufficient quantity for spectroscopic study appears yet to have been isolated.

The greatest gap in the list occurs among the rare earths, and arises from the unrivalled complexity of their spectra. Considerable progress has recently been made at both ends of the group, where the spectra are less intricate, and in the middle, where a relatively simple set of terms is superposed upon the complex mass. Within a few years more, we may hope for notable advances here, though the complete analysis of neodymium or dysprosium (for example) should only be attempted by a young man if he hopes to live to finish it!

For the heavy elements beyond the rare earths progress is greater. The noteworthy success of Rasmussen with radium and its emanation shows that neither costliness nor instability need deter a sufficiently skilful observer.

The analysis of atomic spectra may in general be regarded as well advanced. That of the spectra of diatomic molecules is making excellent progress. Good analyses have been made for many hydrides, oxides and halides, and for other important molecules such as C_2 and CN.

For polyatomic molecules, only a beginning has been made, but the vibrational relations for H_2O, NH_3, CH_4, and some others are well understood, and this suffices to permit the correlation of many groups of bands, and even the prediction of others, though the individual band lines remain uninterpreted. In the determination of energies of dissociation much aid is here available from thermochemistry, and some reactions of astrophysical interest have been carefully studied because they are also of industrial importance.

II. *Astrophysical Applications of this Analysis*

The knowledge of the energy-structure of atoms, which has resulted from spectroscopic analysis, has had far-reaching applications in the theory of atomic structure, in physical chemistry and in other fields, which cannot be described here. But nowhere has it rendered greater service than in astrophysics.

Long before "modern physics" had been born, Sir Norman Lockyer, from spectroscopic data, had divined the dissociation of atoms in the hotter stars—pointing out successive stages—and had shown from direct and simple evidence that there was less sodium vapour in the whole thickness of the Sun's atmosphere than in an inch of Bunsen flame. A generation was to pass before the full implications of his ideas were realized. In the meantime, and while the study was still in its empirical stage, more detailed analysis

had enabled Huggins to point out the importance of ultimate lines, such as *H* and *K* of Ca^+, and Hale and his colleagues to prove, by the use of King's early temperature classification, that sunspots were cooler than the photosphere, and, by the presence of the Zeeman effect, that they were the seats of strong magnetic fields.

The beginning of the new era dates from Saha's classical papers of 1920, in which the application of physical chemistry—the theory of mass-action as applied to ionization and dissociation—opened the way to understanding a host of things that had till then been perplexing. A stellar atmosphere is a complicated, but withal surprisingly comprehensible, example of statistical equilibrium among a multitude of systems—atoms of many kinds, in various stages of ionization and excitation, but all interacting with the free electrons, and with the flood of radiation escaping from below.

The first, and still the greatest, triumph of the new conceptions was the interpretation of the sequence of stellar spectra. The systematic study of thousands of spectra at Harvard had shown that the long recognized and very different types of spectra, like those of Vega, the Sun and Betelgeuse, were connected in imperceptible steps by a continuous series of intergrades, but always in the same fashion, so that they could be arranged in a linear sequence—as is recognized in the familiar Draper Classification, with its decimal divisions. Lockyer had realized that the white stars were hot and the red ones cool, and had explained the disappearance of the metallic arc lines by "dissociation." Saha, in 1921,* defined this more precisely as ionization of the atoms and pointed out that the appearance of the lines of neutral helium only at temperatures high enough to ionize the metals completely was explicable by the high energy of excitation required to get helium atoms into states capable of absorbing the observable lines. He also showed that the degree of ionization might be used to calculate the temperatures of stellar atmospheres, provided that the pressures could be estimated, and suggested that the absolute magnitude effects discovered by Adams and Kohlschütter were probably due to differences in atmospheric density. The later developments of this creative idea have been so numerous and varied that it is more profitable to discuss them by subjects of investigation, rather than in historic order.

The astrophysicist has now at his service :

(*a*) A detailed quantitative theory, well tested by observation, which enables the precise calculation of the distribution of the atoms among their various states of ionization and excitation, in any mass of gas in thermodynamic equilibrium, given the composition of the mixture, its temperature, and either the total pressure or that arising from the free electrons. A simple modification of the equations takes account of the case where the flux of radiation passing through the gas is black-body radiation corresponding to some other temperature, and with any given factor of dilution.

(*b*) For the large majority of spectra, precise quantitative data are available regarding the energies of ionization of the atoms, and of excitation of dozens, or often hundreds, of spectroscopic levels, and a specification of the

* *Proc. Roy. Soc.*, A, **99**, 135, 1921.

transition involved in the production of each important line. The statistical weights of the levels are also known. The "*f*-factors" which define the probabilities of the various permissible transitions from a given level are known absolutely in a few cases—as for the Balmer series—while their relative values for the lines of a multiplet, and for groups of supermultiplets, are given by formulæ which, though based on approximate theory, are good for the individual lines of the lighter elements, and probably for heavier atoms as a statistical average. The Zeeman and Stark effects may also be calculated, when desired, for any given line.

(*c*) Finally, useful, though by no means definitive, formulæ exist for deriving the general opacity of a layer of a wholly or partly ionized gas, for radiation intermediate between its absorption lines.

With these precise and powerful tools, problems of varied types have been successfully treated.

Let us consider first the identification of solar (or stellar) lines—that is, the qualitative analysis of the "absorbing" layers. The classical method here is by coincidences of wave-length. When the solar lines are numerous (as for iron) or strong (as for sodium or hydrogen) this gives conclusive results, but the case is quite different for elements which are represented by a few weak lines.

Rowland's Tables record 16389 lines between λ 3000 and λ 6000. There is one chance in nine that a "wave-length" written down at random will agree within 0·01 A. with a solar line; * for a hundred such fictitious positions, we should anticipate 10 ± 3 coincidences as close as this if the solar lines were uniformly distributed (and, in fact, a little more).

Merely numerical coincidences are therefore of little weight unless there is some check upon them. The obvious test is that, unless the stronger lines are present, coincidences for weaker ones are without significance. But this must be applied with some care, for the conditions of excitation in solar and laboratory spectra may be very different. In the former case the number of atoms in excited states falls off very rapidly with increasing excitation potential, and lines absorbed from highly excited states are weak. In the arc, however, re-combinations of ions and electrons steadily replenish the higher states, and the corresponding lines may be much stronger. A good example is found in copper.

		Intensity	
λ	E.P.	Kayser and Runge (arc)	Rowland (Sun)
3247	0·00	10 R	10
3273	0·00	10 R	10
5105	1·38	8 R	4
5782	1·64	8	6
5153	3·77	8 R	1
5218	3·80	10 R	1

* H. N. Russell and I. S. Bowen, *Ap. J.*, **69**, 202, 1929.

The critical lines are those arising (in absorption) from the ground-state of the atom or one close to it, and having large transition probabilities. These atoms appear, whether in the laboratory or in celestial sources, when the element is present in but small proportions—they are de Gramont's *raies ultimes*. When they are in the accessible spectral region, and are not masked by accidental coincidences with lines of other elements, the spectroscopic test is very delicate. The lines of the same multiplet, or group of related multiplets, come from substantially the same energy-level. If the weaker ones are really present the stronger components must be there too—or else be masked. In this way the relative importance of the contributions of two or more elements to a blended solar line may often be determined.

Recognition of the rôle played by excitation potential has almost completely cleared up the once puzzling "absences" of many elements from the solar spectrum. The ultimate lines of all the non-metals, and of some metals like gold and mercury, lie beyond λ 3000 and are inaccessible in celestial sources. For a number of these—carbon, nitrogen, oxygen, phosphorus and sulphur—the "penultimate" lines are in the deep red or infra-red. The first three show fairly strong solar lines in this region, the others weaker ones. Boron and fluorine, for which this method fails, have been detected by their compounds. Most of the other outstanding "absences"—*e.g.* neon and argon—are completely explained by the high excitation potential of their accessible lines. Other apparent failures arise from too easy ionization—the neutral atoms being so depleted that their ultimate lines do not appear, while those of the ions are of too short wave-length. At the lower temperature of sunspots, lithium, rubidium and indium, appear faintly; but cæsium is missed out altogether. The few lines attributed in the "Revised Rowland" to Cs^+ have such high excitation potentials that it is now certain that the coincidences are accidental.

The only real absences (of ultimate lines) are for some of the heaviest and rarest elements—rhenium, bismuth and radium.

The same principles of analysis have identified many solar lines not yet observed in the laboratory. The position of the fainter components of multiplets may be predicted with extreme precision from their frequency-differences from known lines, and in this way hundreds of lines of iron and other metals have been identified in the Sun with certainty.

There is an unusually interesting line at λ 6155·148, to which Rowland assigned the intensity 7—though 5 would be a better estimate. During the revision of his Table in 1928 no identification could be found for this line despite its considerable intensity. It is considerably weakened, but not obliterated, in sunspots. This suggested that the line arose from a highly excited state of some neutral atom. Its strength, under these conditions, indicated a very abundant element. The spectra of all the abundant elements had already been thoroughly investigated, except for silicon, so that it became probable that this was a silicon line. In 1933, Kiess observed many new lines in the silicon arc, one of which agreed perfectly with the solar line, and had the very high E.P. of 5·59 volts.

In the infra-red, the identification of solar lines is peculiarly difficult.

The solar measures are precise, but the laboratory spectra have perforce been taken with lower dispersion, and often with powerful light sources, such as heavy current arcs, which give diffuse lines. The standard of tolerance for these measures is so wide that secure identification can hardly be made from wave-lengths alone—except for iron, which is even more predominant here than in the visible. Some important multiplets, however, involve terms whose energy-differences are precisely known from lines in other spectral regions. When the corresponding differences in wave-number are closely reproduced among the infra-red solar lines, and the relative intensities agree with expectation, the presence of the multiplet is assured. Two series of magnesium lines, separated by a constant frequency-difference, have thus been found in the Sun, though only a part of one of them has been detected in the laboratory by Paschen.

The distinction between lines arising in the Sun's and in the Earth's atmosphere can be made with certainty by comparing spectrograms of the Sun at different altitudes, or of the east and west limbs. Band-spectrum theory has been beautifully applied (by Babcock and Hoge *) in the identification of molecules containing the rare heavy isotopes of oxygen.

When all these new tests have been applied, there remain a great number of faint solar lines for which no identification has yet been made. Many of these may belong to spectra not yet fully investigated, such as the rare earths ; others may belong to abundant elements like iron, and be identified by prediction, from multiplet relations ; but there appear to be far too many of these lines to be accounted for thus. It is probable that many—perhaps most—of them belong to the band-spectra of refractory molecules, which exist in small amounts in the Sun's atmosphere.

The identification of stellar lines is complicated by the lower accuracy of the wave-lengths and the greater number of unresolved blends—though the great coudé spectrographs largely remove these difficulties. The principles already discussed have been equally fruitful, but there is a good deal more to do.

Next in order comes naturally the quantitative analysis of solar and stellar atmospheres. A good idea of the relative order of abundance can be derived empirically. The spectrum of iron is more fully developed in the Sun than it can be obtained in the laboratory even by special effort. For titanium, the laboratory spectrum is definitely richer in lines ; for cobalt, or copper, much richer ; while for silver only the ultimate lines appear in the Sun, and faintly at that.

Numerical determinations were made possible by the definite equations of the ionization theory. The first successful study by Fowler and Milne made use of the maxima of intensity of various spectral lines, in the sequence of the Draper classification, to determine the effective pressure and then of the marginal appearances and disappearances of these lines, as the temperature and ionization varied from star to star, to determine the relative abundance of the elements concerned. Two very important conclusions followed. Firstly, the electron pressure in stellar atmospheres is in general extremely

* *Phys. Rev.*, **37**, 227, 1931.

low—of the order of 10^{-4} to 10^{-6} atmospheres. Secondly—as Miss Payne showed shortly afterwards—the stars, taken as a whole, are remarkably alike in chemical composition. The differences between the spectra of various classes are caused almost entirely by differences in the temperature and density of the reversing layers. There are some exceptions—notably the N and R stars, which actually contain an excess of carbon, as R. H. Curtiss suggested years ago—but these are rare enough to emphasize the general similarity.

Postponing for the moment further discussion of this ingenious and powerful method of research, we may consider another method of quantitative analysis—by the widths and intensities of the spectral lines. On classical theory the width of "absorption" lines (which are actually produced by scattering) should be proportional to the square root of the number of active atoms per unit cross-section of the scattering layer. This was first applied to the Sun in 1924 by J. Q. Stewart,* who estimated the number N of atoms active in producing one of the D lines as 3×10^{16} per cm.^{2}. Four years later, Unsöld,† extending the theory somewhat and measuring the profiles of a number of strong solar lines, derived values of N ranging from $1{\cdot}5 \times 10^{19}$ for the K line to 2×10^{14} for the *Sr* line $\lambda 4607$. The former corresponds to a layer of gaseous Ca^+ at standard pressure and density only $5\frac{1}{2}$ millimetres thick, yet this gives the strongest line in the whole solar spectrum.

This method demands great resolving power, and is applicable only to the stronger lines. The measurement of "equivalent widths" (which are relatively little influenced by the resolution) should extend this method, but such measures have so far been made only in selected regions and for a small fraction of all Rowland's lines. Rowland's intensities, though no more than rough estimates on an empirical scale, can be made very useful by proper calibration. This was done by Russell, Adams and Moore with the aid of the theoretical relative intensities of the lines in 228 multiplets of many elements. The results, though decidedly rough for individual lines, should give good average values. Simple addition then gave the numbers of atoms of a given element which were at work in producing its observable lines. Allowance for the unobservable lines in the infra-red and ultra-violet was not so difficult as it sounds. So long as all the strongest multiplets from one spectroscopic term are accessible, the number of atoms in the corresponding energy state can be found. Application of the quantum weights and Boltzmann factors determines the rest. Neutral and ionized atoms are determined separately; in some cases both are available for the same element, the results indicating an effective electron pressure of $2{\cdot}5 \times 10^{-5}$ atmosphere. The final outcome was a quantitative analysis of the Sun's atmosphere for 56 elements.‡ The similarity to the composition of the Earth is conspicuous. The rock-forming elements *Na*, *Mg*, *Al*, *Si*, *K*, *Ca*, *Fe* are the most abundant in both cases. There is a steady drop in abundance with increasing atomic number—slowing up for the heavier

* *Ap. J.*, **59**, 30, 1924. † *Zs. f. Physik*, **46**, 765, 1928.
‡ H. N. Russell, *Ap. J.*, **70**, 11, 1929.

atoms—and the well-known predominance of even atomic numbers over odd is conspicuous. Only a few elements stand out from the general trend. Iron is exceptionally abundant, and scandium and gallium rare. The most remarkable irregularities are found for the light atoms. Lithium and beryllium are exceedingly rare, being present in about a millionth of the amount suggested by a uniform change with atomic number. It is more than probable that this arises from the instability of their nuclei (now demonstrated in the laboratory), and that these atoms, in the hot interiors of the stars, are actually broken down or built up into others.

Hydrogen is excessively abundant. The Balmer lines, despite the great E.P. of 10·15 volts, are very strong, beyond the limit of reliable calibration. Oxygen is apparently next in abundance, though far behind hydrogen. Helium may rival it.

Miss Moore's thorough discussion* of the sunspot spectrum has shown that the differences from that of the disk can be quantitatively explained by the influences of lower temperature, lower ionization and electron pressure, and the increased transparency of the gases which results from this. All the effects characteristic of the spot—whether thermal (intensities), magnetic (Zeeman effect), or radial velocity (Evershed effect)—diminish towards the violet, and for strong lines relative to weaker ones. Part of these changes are due to scattered photospheric light and to other blending effects.† The magnitude and causes of the residuum deserve active investigation during the approaching spot maximum.

It is to be hoped that new calculations, based on accurate measures of equivalent widths, still perhaps calibrated by multiplets, will ere long replace these preliminary data; but the possibility of obtaining so much from Rowland's intensities shows the value of estimates on even a roughly homogeneous scale. The astronomer familiar with the value of the ancient system of estimated stellar magnitudes, especially when photometrically calibrated, will probably be less surprised by this than the average physicist.

Analyses of some stellar spectra have been made on the same principles by Adams and Russell,‡ who assumed provisionally that the solar calibration could be applied to the stars. Struve has since shown that the gradients of intensity for lines of the same multiplet may be very different in different stars. Because of this "turbulence" effect, the conclusion of the earlier paper regarding departures from thermodynamic equilibrium should be taken with much reserve, until each spectrum has been independently calibrated with its own multiplets.

The whole amount of gas, with the exception of hydrogen, which is effective in producing the solar absorption lines, was found to be 4×10^{21} atoms/cm.^{2}, equivalent to a layer of gas five feet thick at standard temperature and pressure. The Sun's atmosphere must be many miles thick; hence its density and pressure must be very low. Why we do not see deeper was first pointed out by Stewart,§ who showed that an ionized gas must

* *Ap. J.*, **75**, 222, 298, 1932.
† R. B. King, *Ap. J.*, **80**, 136, 1934.
‡ *Ap. J.*, **68**, 9, 1928.
§ *Phys. Rev.*, **22**, 324, 1920.

necessarily scatter light and behave as if it were slightly foggy. More detailed investigations by Milne, Gaunt, Pannekoek and others have fully confirmed the reality of this type of opacity, and shown that the greater part of it arises from the photo-electric ionization of excited atoms. This reveals itself in the spectrum by continuous absorption starting at the head of spectral series and fading out gradually toward shorter wave-lengths. In the A stars the absorption beyond the heads of the Balmer and Paschen series is directly observable. In spectra of the solar type a great many energy-states of excited metallic atoms co-operate. Woolley has recently shown, from a detailed discussion of these,* that the approximate constancy of the opacity coefficient throughout the visible spectrum is explicable, though with some difficulty.

The success of these interpretations is the more remarkable, when it is realized how egregiously simplified are the postulates on which they are based. A stellar atmosphere is treated as a layer of gas at constant temperature and pressure superposed upon a "black" radiating surface of the same temperature. The real atmosphere is bounded above by a vacuum and below by a fog, and has no definite boundaries; its temperature increases downward, though by but a moderate percentage, and its pressure diminishes practically to zero at the top. This drastic over-simplification, however, works, and works admirably, in practice. The reasons why it does have been found by several investigators. The atmosphere, though varying in temperature from point to point, is subjected to a flood of radiation which has approximately the quality of black-body radiation corresponding to the effective temperature of the photosphere. Pannekoek has shown that it is this photospheric temperature which appears in the all-important exponential terms in the equations for ionization and excitation. The local temperature has quite a subordinate influence, and may be sufficiently allowed for by changing the value of a numerical constant.

The pressure, and the degree of ionization, vary very greatly with the depth. But Milne has proved that, in an isothermal atmosphere in equilibrium under its own weight, the ratio of the total numbers of neutral and ionized atoms of any element above a given level will be nearly the same as if the electron pressure were uniform throughout, and one-half the value at the bottom. In a number of simple cases this relation is exact, and in all instances of practical importance its errors may be neglected.

The assumption of a definite atmospheric depth is also justified by Milne's researches, which indicate that the profile of a dark line formed by the contribution of atoms at different optical depths in an atmosphere of increasing haziness will be similar to that produced in a layer of gas, transparent except for selective absorption and scattering, and containing as many atoms of the kind considered as actually occur above an optical depth τ varying from one part of the line to another, but averaging somewhat less than 0·5.

The quantity of gas above this depth depends on the coefficient of general opacity, k, and hence upon the pressure, the temperature and the atomic

* *M.N.*, **95**, 101, 1935.

composition. In Fowler and Milne's pioneer work, k was assumed to be constant—leading to a fixed amount of matter above the photosphere. The predicted effects of temperature agreed with observation for the stars of Class B and again for those cooler than Class F, though the H and K lines, and some others, were discordant, but the behaviour of H, Mg^+ and Fe^+ in the A stars was not represented. The influence of absolute magnitude—that is, of surface gravity—was not satisfactorily explained.

It now appears, as a result of the work of several investigators, that the success of the original theory was due to a fortunate compensation of errors. Later theoretical work shows that k varies as $PT^{-\frac{3}{2}}$, where P is the electron-pressure. At low temperatures the opacity for a given electron-pressure increases rapidly ; but the degree of ionization rapidly decreases. For an atmosphere similar in composition to the Sun's the two effects nearly neutralize one another, and the amount of matter above a given optical depth fluctuates, without much systematic change, till the hydrogen begins to become ionized. Since hydrogen is in great excess, the opacity then increases rapidly and "the photosphere rises." When once the hydrogen is fully ionized, say in Class B8, second ionization of less abundant elements can do little in proportion, and the "extent" of the atmosphere changes slowly. It is the increasing opacity due to ionization of hydrogen, and not the second ionization of the metals, which leads to the maxima of Fe^+ and Ti^+ in the later divisions of Class F. Conversely, the existence of residual opacity due to ionized metals is the main factor in fixing the maximum of the Balmer series.

Though the theory of maxima thus becomes more complicated, it presents no serious difficulties. To the first approximation, the problem of accounting for the various types of stellar spectra may now be regarded as satisfactorily solved. The previously recalcitrant cases now fall into line, and the absolute magnitude effects are also accounted for. The values of surface gravity, and their variation in giants and dwarfs of different spectral classes, must of course be taken from observational data, as must the composition of the solar atmosphere. Apart from these no adjustable empirical constants are required. In my work of 1933, one such constant was introduced, but Professor D. H. Menzel kindly permits me to say that he and Mr. C. L. Pekeris have found that if the quantum weights now known to be correct are introduced into the fundamental formulæ, the agreement is satisfactory without any such adjustment.

More recent work by Cambresier and Rosenfeld, and of my own, shows that the abundance of molecules in stellar atmospheres can be treated with equally satisfactory results. The weakening of the *CH* and *CN* bands in the M stars, and the sensitiveness of the latter, but not of the former, to absolute magnitude, are well represented. The abundance of hydrides among the compounds confirms the great abundance of hydrogen in stellar atmospheres. There appear to be at least twenty times as many hydrogen atoms as of all other kinds together.

The differences between Classes M, N, and S must be attributed to differences in chemical composition. Berman's careful study of R Coronæ

Borealis * has detected a star in which carbon is in excess, though the temperature is too high to permit much formation of its compounds. One or two prominent characteristics of stellar spectra remain to be accounted for. The most important of these is doubtless the recently discovered division of the Wolf-Rayet stars into two sequences, similar in range of excitation, but characterized by bright bands of carbon, or of nitrogen, to the exclusion of the other. The great strength of enhanced metallic lines in red super-giants like Antares is probably to be explained by the existence of outer atmospheres, so extensive that they can no longer be approximately represented by a layer of constant temperature and pressure. Direct evidence of an enormous envelope of this sort has been found in Zeta Aurigæ.

While the problem is solved in the first approximation, the interpretation of the fine details of stellar spectra promises an abundance of difficult and interesting problems.

The theory of the profiles and central intensities of "absorption" lines is still imperfect. The earlier forms suffered from the serious disadvantage that they all predicted that the centres of the stronger lines should be quite dark—in other words, that spectroheliograms should show nothing. Recent work removes this difficulty, but the formulæ of different investigators still disagree.

The widths of strong lines are mainly due to damping and are proportional to $\sqrt{N}$, where N is the number of atoms effectively at work. For faint lines, the Doppler broadening is predominant, and the equivalent width is proportional to N. There is an intermediate range in which the change with N is small. The magnitude of both damping and Doppler effect is often many times greater than is indicated by classical theory. Turbulence in the atmospheres may account for much † of the latter (as Struve has suggested) and a more detailed wave-mechanical study of the mean lives of high spectroscopic levels may help to understand the damping ; but much more study, observational and theoretical, has yet to be made.

Our knowledge of the transition probabilities, upon which the intensities of spectral lines primarily depend, is still very imperfect. For the simple spectra, much can be done theoretically ; for complex ones, such as the very important spectrum of iron, there seems no present hope of this—indeed, the approximations so far available do not even predict the existence of inter-system combinations, one of which is the great line $\lambda 4383$. An accurate observational determination of intensities in the spectrum of neutral iron is greatly to be desired, and appears to be practicable, though not exactly easy. At present we know the wave-lengths to seven significant figures and the intensities hardly to one. When this becomes available the intensity scales (measured or estimated) in stellar spectra can be precisely calibrated, the temperatures determined from the relative strength of lines of different excitation potential, and a far more thorough analysis of the atmosphere made than is now possible.

Bright lines in celestial spectra are much harder to explain than dark

* *Ap. J.*, **81**, 369, 1935.

† See the admirable summary by Minnaert, *Observatory*, **57**, 328, 1934.

lines. The best understood of these at present are those which were most mysterious a decade ago—the gaseous nebulæ. The solution came with Bowen's brilliant discovery that the perplexing lines arise from forbidden transitions between terms of the same parity in familiar atoms. These have time to happen only in an excessively rarefied gas. The elements whose lines appear are just those which might have been expected—the permanent gases, which condense only at very low temperatures. Only a few faintish sulphur lines are exceptions to this rule.

The stratification of radiation in the planetaries, with the lines of highest excitation nearest the centre, results from the process of fluorescent excitation by light from the intensely hot nuclei. From a study of this process, Zanstra has obtained quantitative estimates of the temperatures of the nuclei, while Bowen has shown that the apparently capricious appearance of a few permitted lines of *O*III and *N*III results from the excitation of a particular energy-state of one atom by the absorption of light emitted by atoms of a different kind. The forbidden lines of *Fe*II which appear in η Carinæ and some other stars suggest a denser medium.

The expanding shell of gas ejected from a Nova produces a nebula of diminishing density. Boyce, Menzel and Payne have followed the emission of permitted lines in the early stages, then of the weakly prohibited "auroral" lines, and lastly of the less probable "nebular" transitions. Wave-mechanical calculations have verified the correctness of their interpretation.

The Wolf-Rayet stars present expanding envelopes of smaller dimensions, but still nebular rather than stellar in character. The simultaneous presence of lines corresponding to widely different degrees of ionization and excitation confirms this, while the appearance of dark lines of *N*v indicates a surface temperature of 80,000° for some of the central stars—the highest yet definitely assignable. Spectra resembling P Cygni appear to represent a less violent repulsion of the atmosphere. The ordinary Be stars are not so well understood.

The bright lines in long-period variables are very puzzling; conditions in the outer parts of these enormous and tenuous bodies appear to be very complicated.

Perhaps the most puzzling bright lines of all are those which appear in many otherwise well-behaved dwarfs of Class M. The companion of Castor, in particular, shows the Balmer lines and H and K bright. It is an eclipsing binary, with components of about 70 per cent. the Sun's size and 60 per cent. of its mass. The gap between the components is less than the sum of their radii, yet the bright lines of the two are strong and sharp—which appears inconsistent with the existence of any extensive envelope. It is hard to see, too, how energy enough to excite the hydrogen atoms can be fed into them at the low temperature.

A few stellar lines are now known to be produced by absorption in a medium far more rarefied than the nebulæ—scattered atoms distributed in interstellar space. Only ultimate lines, of course, can be expected, and those of neutral sodium and ionized calcium are observed. Eddington has

accounted fully for their behaviour—the absence of neutral calcium and the visibility of the sodium lines despite the low ionization potential. Few other possibilities appear promising. The ultimate lines of magnesium and silicon are out of reach, those of iron are too numerous to permit any single one to be strong ; while potassium is so easily ionized that few of its atoms should be neutral. It might be worth while to hunt for the red lithium line—not that it is likely to be of observable strength, but because, if it should be, we could be morally certain that these lithium atoms represented really primitive matter and had never been inside the stars, and subject to nuclear disintegration.

Merrill * has recently discovered four apparently interstellar lines of unknown origin. The ultimate lines of both neutral and ionized atoms in the visible region are now so thoroughly known that it appears probable that these lines, if really interstellar, are absorbed by molecules of some kind. If the temperature of the gas is low enough—more precisely, if the rotation of the molecules is little excited—only a few of the band lines observed at room temperature might appear.

We may finally return from interstellar space to our own system, and from hot bodies to cold ones. The luminosity of comets presents a considerable similarity to that of nebulæ, being a fluorescent emission excited by solar energy. In this case, however, we have to deal with simple resonance radiation of the ultimate bands absorbed by the molecules in their unexcited states. The total energy emitted is small enough to be consistent with the perfect transparency of Halley's Comet when transiting the Sun in 1910. The fact that N_2 and CO appear only in the ionized state follows from the position of the ultimate bands of the neutral molecules in the far ultra-violet—but we have no adequate explanation how solar radiation at the Earth's distance can provide the very high energy of ionization of these molecules.

In the planets we have to deal with enormously greater thicknesses of gas than anywhere else in astrophysics. The main absorption bands of constituents of our own atmosphere completely obliterate great regions of the spectrum, and bar the most interesting part of the ultra-violet, and great regions in the infra-red, hopelessly from our knowledge. The strong telluric oxygen bands are millions of times weaker in proportion to the number of molecules involved, than are ordinary atomic lines, and correspond to a truly forbidden (quadrupole) electronic transition. The telluric bands of water-vapour, and those of other gases which have recently been identified in the planets, belong to the oscillation-rotation systems of these molecules, and represent higher harmonics or combinations of the fundamental vibration-frequencies. Spectroscopic theory has been of conspicuous aid in their identification. For example, the band near $\lambda 7820$ in Venus showed a distinctive rotational structure from which the moment of inertia of the molecule could be determined as $70{\cdot}5 \times 10^{-40}$ c.g.s. units,† which agreed so closely with the value for carbon dioxide, determined from

* *P.A.S.P.*, **46**, 206, 1934.

† Adams and Dunham, *P.A.S.P.*, **44**, 243, 1932.

bands in the far infra-red, as to make this identification practically certain, even before it was confirmed in the laboratory.

The initial identification of ammonia in Jupiter and Saturn was made by the old method of coincidences (upon ample evidence). The same is true in part of methane, though harmonic relations were employed tentatively by Wildt in his original discussion. The later proof, by Slipher and Adel,* that the whole system of huge bands which dominates the spectra of Uranus and Neptune is due to methane depends primarily on the analysis of the vibrational harmonics, and is as beautiful an application of spectroscopic theory as one could desire to see.

The proof that methane and ammonia are just the atmospheric constituents which should be expected to form if a mass of gas, similar in composition to the Sun's atmosphere, should be allowed to cool gradually to a temperature below $-100°$ C., takes us to the boundary where astrophysics merges into physical chemistry, and possibly beyond it.

This has been but a fragmentary survey of a field which has, within little more than a decade, become one of the most fertile in the domain of physical science. There is still a good deal of exploration to be done in the less accessible parts ; and many workers must labour for years before the whole area is adequately exploited. The fascination of the work, and the value of its products, commend it to newcomers, as well as to those who have known it from the pioneer days.

THE SPEED OF RECESSION OF THE GALAXIES.

Sir A. S. Eddington, F.R.S.

1. In November 1931 I gave the results of a first calculation of the limiting speed of recession of the galaxies by a combination of relativity theory and quantum theory.† Since then I have tried to develop the theory more fully. I have now been able to reach a solution of the problem which I regard as definitive. The result is 865 km. per sec. per megaparsec.

The actual speed of recession is less than the limiting speed owing to the restraining effect of the gravitational attraction between the galaxies. It seems rather unlikely that this makes any large difference. To reduce the recession below 750 km. per sec. per mp. it would be necessary to assume a mean density of matter in the universe greater than 10^{-28}. The result therefore suggests that the present scale of distances of the remote nebulæ is rather too great and should be reduced by 20–30 per cent.

2. The new investigation appears to decide an ambiguity in the ordinary theory of the expanding universe. It shows that the actual mass M must of necessity be equal to the Einstein mass M_e. Usually three cases are dis-

* *Phys. Rev.*, **47**, 651, 1935.

† *M.N.*, **92**, 3, 1931. The result there given was 528 km. per sec. per mp.

cussed, according as M is greater than, equal to, or less than M_e; and (leaving out æsthetic considerations) it has hitherto been impossible to decide between them. Before giving the reason for the conclusion $M = M_e$, it is necessary to say a little about the theory itself. The details of the actual calculation are highly technical and will be published elsewhere; but the principle of the calculation is not very difficult, and can be stated briefly.

It is evident that the curvature of space-time introduced in relativity theory is equivalent to the waves of "ψ" introduced in wave mechanics. For both devices are used for the same purpose, viz. to represent the energy (or mass) and momentum of physical systems. Both are *devices*; it is not suggested that either the curvature or the waves exist in a literal objective sense. The method of waves gives the finer analysis, and is essential if we are treating systems on the atomic scale; but for the mechanics of ordinary macroscopic objects and for astronomical and cosmical systems either method is available, and either method should, of course, give the same result.

But the result will be expressed in terms of different natural constants. When the relativity method is used the principal natural constant is the constant of gravitation κ. Wave mechanics does not bring in κ, but involves constants such as h, e, m. If then we can find *one* problem tractable enough to be solved by both methods, we shall obtain a relation between the natural constants by comparing the two answers. This fundamental relation is evidently of the highest importance. I cannot but think that the realisation that such a relation must exist—that there will be at least one redundant constant when the two theories are brought together—is scarcely less important than the ascertainment of its precise form.

It does not matter how artificial the problem is. In determining the constant of gravitation observationally, the physicist is not forbidden to arrange the matter in his laboratory in any way that suits the experiment; so also the theoretical physicist is not forbidden to arrange the matter of the universe in any way that makes his calculation easier. In either case the value found for the constant will apply to all possible distributions of matter—differing widely from those used in the experiment or calculation. Only we must take note that, in rearranging the matter to suit his purpose, the experimenter cannot, and the theorist must not, violate any law of nature.

3. I now put the following question: Under what circumstances can a system of N particles remain static, assuming that no radiation is present?

The answer of wave mechanics is that the system must be in its "ground state." Using the exclusion principle, we can work out a relation between the energy (or mass) and volume of the system by the method now familiar to astronomers through its application to degenerate matter in white dwarf stars.

The answer of relativity theory is that the system must be an Einstein universe. The relation between mass and volume is given by a well-known formula of the theory.

Here then is a problem which can be treated by both methods. It has not been easy to obtain the wave mechanics solution, owing to the many side-questions which are raised. For example, it was necessary to decide which

is the right degeneracy formula ; * and the implicit assumptions in the elementary equations which contain the definition of mass, as the term is employed in wave mechanics, must be brought to light, in order to ascertain the precise application of the formulæ. But in the end the difficulties have resolved themselves.

On one important point the answer of wave mechanics is more explicit than that of relativity theory. The latter says that *if* there is a static arrangement of the N particles it will be an Einstein universe ; but there will be no static arrangement unless $M = M_e$. The former says that there is a static arrangement, viz. the ground state, in all circumstances. To reconcile the two answers $M = M_e$ must be a necessary relation. The detailed theory shows how this adjustment comes about automatically.

4. A by-product of the investigation is the number N of elementary particles in the universe, *i.e.* $\frac{1}{2}N$ protons and $\frac{1}{2}N$ electrons. I have previously suggested that the number N which occurs in the formulæ is an extrapolated value of the number of particles—calculated for a spherical model and only roughly applicable to the actual universe.† But according to the new results N is in all cases the actual number. (Positrons and negatrons are counted as *minus* electrons and protons.) This gives a special interest to the actual value of N. I have therefore taken special precautions in calculating its value from the observed constants. The result is $N = 1{\cdot}573 \,.\, 10^{79}$. It should be trustworthy to about 1 or 2 parts in 1000 ; the uncertainty is mainly in the constant of gravitation, for which I have adopted $6{\cdot}664 \,.\, 10^{-8}$.

It has been suggested by R. Fürth that the factor mainly responsible for the large value of N is 2^{256}. Following up this hint, we find that the above value of N is

$$N = 135{\cdot}82 \,.\, 2^{256}.$$

As a rule it is unprofitable to try to guess the composition of a natural constant from its observed value ; there are too many plausible suggestions which give a sufficiently close result. But N is rather exceptional, because it is an integer by definition. No transcendental or irrational factors can enter into its composition, and the resources of the numerologist become thereby very limited. The integers concerned in N are presumably those which are prominent in the theory of the double wave vector, which is the starting-point of general relativistic wave mechanics in the same sense in which the interval is the starting-point of the ordinary general relativity theory. In view of the particular association of the numbers 256 and 136 with the double wave function, it seems probable that the exact value of N is $136 \,.\, 2^{256}$.

* *M.N.*, **95**, 194, 1935.
† *New Pathways in Science*, p. 254, 1935.

OBSERVATIONS OF NOVÆ, 1934–1935.

W. H. Steavenson.

The observations which follow were all made with the 20½-inch Hindle reflector.

In the case of novæ already dealt with in previous papers, references to the comparison stars used will be found in *M.N.*, **92**, 720, 1932.

The magnitudes of the stars used for Nova Herculis are from *H.A.*, **50** and **54**, and were circulated with a binocular chart to members of the Variable Star Section of the B.A.A.

Charts of faint stars near Nova Aquilæ (1925) and Nova Lyræ (1919), with my estimates of their magnitudes, are now given for the first time, since these objects are now so faint as to be difficult to identify from small-scale maps published when they were bright.

Nova Aquilæ (1918)

Observed on six nights. The general fading has now ceased for some years, and there is very little variability in the light of the nova, at any rate as a visual object.

Date	Mag.	Date	Mag.
1934 May 11	10·7	1934 June 29	10·8
31	10·7	July 3	10·8
June 9	10·8	Aug. 29	10·7

Nova Aquilæ (1925)

Observed on three nights. This nova rose to a maximum (visual) of $8^{m}\cdot7$ in 1925 October. It was not found on earlier photographs showing stars down to magnitude 16·5, and it might have been expected, after the lapse of nine years, to have passed again below this limit. In 1934, however, it was found still to be brighter than the fifteenth magnitude, which indicates an unusually slow fall, if the nova is indeed still fading.

Fig. 1 shows the position of this object in relation to the nearest B.D. star. My estimates of the magnitudes of a few of the neighbouring faint stars are as follows :—

A = 14·0	E = 14·5
B = 14·3	F = 14·5
C = 14·4	G = 14·8
D = 14·4	H = 15·0

On 1934 July 6 and 14 the nova was estimated to be of mag. 14·8, and on August 3, 14·9.

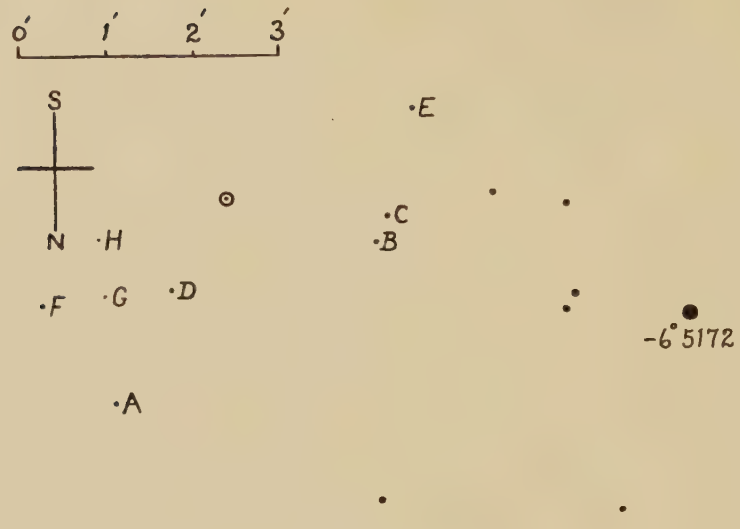

Fig. 1.—*Nova Aquilæ* (1895).

Nova Aurigæ (1891)

Observed on three nights. This nova is slightly variable, but the total range, as observed over a period of many years, has not exceeded $0^m.3$. The image is sensibly stellar in character.

Date	Mag.
1934 Oct. 20	15·0
31	15·0
Dec. 5	14·9

Nova (*T*) *Coronæ* (1866)

Observed on one night, 1934 May 18, when the magnitude was 9·96. The nova has been of this brightness, without perceptible variation, for a number of years. It is orange-yellow in colour and in this respect is unlike the majority of faded novæ.

Nova Cygni (1876)

Observed on fifty nights. This object is slightly but definitely variable. The extreme range in 1934–35 amounted to $0^m.5$ (14·6–15·1). There is nothing peculiar about the appearance of the nova as seen in the reflector.

Date	Mag.	Date	Mag.	Date	Mag.
1934 May 5	14·86	1934 June 13	14·91	1934 July 4	14·76
11	14·91	16	15·00	5	14·76
17	14·91	17	15·10	6	14·81
31	14·81	27	14·81	7	14·81
June 9	14·86	July 1	14·61	8	14·76
12	14·91	3	14·81	9	14·76

Date	Mag.	Date	Mag.	Date	Mag.
1934 July 10	14·71	1934 Aug. 11	14·66	1934 Aug. 31	14·71
11	14·76	12	14·71	Sept. 1	14·81
14	14·76	15	14·76	3	14·81
16	14·81	18	14·71	4	14·76
19	14·76	19	14·81	9	14·81
27	14·76	20	14·76	10	14·91
30	14·81	25	14·71	13	14·76
31	14·81	26	14·71	16	14·81
Aug. 3	14·71	27	14·76	18	14·86
4	14·81	29	14·71	Oct. 5	14·91
10	14·66	30	14·66		

Nova Cygni (1920)

Observed on five nights, 1934 May 5, 11, 17, June 13 and July 3. On each occasion the magnitude was estimated to be 15·4. There has been no appreciable fading of the light since 1933, and a steady state seems to have been reached, with little or no variability. Before its outburst in 1920 the nova was fainter than the sixteenth magnitude (photographic).

Nova Herculis (1934)

Fifty-two observations are given below, down to 1935 April 14. Further observations have since been made, but as they depend on comparison stars for which reliable magnitudes are not yet available their publication has been deferred.

Date (U.T.)	Mag.	Date (U.T.)	Mag.	Date (U.T.)	Mag.
1934 Dec. 13·64	2·5 ±	1935 Jan. 15·71	2·7 ±	1935 Mar. 14·00	4·20
13·72	2·8	19·76	2·2 ±	16·03	4·20
15·87	3·02	24·73	2·4 ±	17·98	4·5 ±
16·75	2·99	25·74	2·42	19·03	4·61
17·81	2·86	27·74	2·42	20·96	4·61
18·72	2·64	31·74	3·58	21·02	4·86
20·73	2·5 ±	Feb. 3·01	3·35	22·00	4·91
23·73	2·12	7·74	2·99	23·01	4·63
26·90	2·8 ±	9·02	2·80	26·00	3·90
27·73	2·61	17·01	3·50	28·05	4·30
29·76	3·29	17·97	3·2 ±	Apr. 3·01	6·51
30·05	3·40	27·04	3·12	5·96	8·32
31·74	2·42	Mar. 4·02	3·84	10·87	9·2 ±
1935 Jan. 4·74	2·70	5·01	3·99	11·94	9·24
5·74	2·8 ±	5·97	3·64	13·00	9·47
10·71	2·9 ±	7·97	3·59	14·92	9·67
12·73	2·42	12·01	4·09		
14·68	2·4 ±	12·98	3·89		

Nova Lacertæ (1910)

Observed on three nights, 1934 May 11, July 3 and August 29. On each occasion the magnitude of the nova was estimated to be 14·4. There has been no sensible change in the light of this object for many years past.

Nova Lyræ (1919)

Observed on two nights, 1934 July 5 and 10. On both occasions the magnitude was estimated to be 14·9. The nova was of sensibly the same brightness in 1925 July, less than six years after its outburst in 1919 December. Unless, therefore, it had a large positive colour-index previous to this latter date, we have here an example of a nova which has settled down (at least

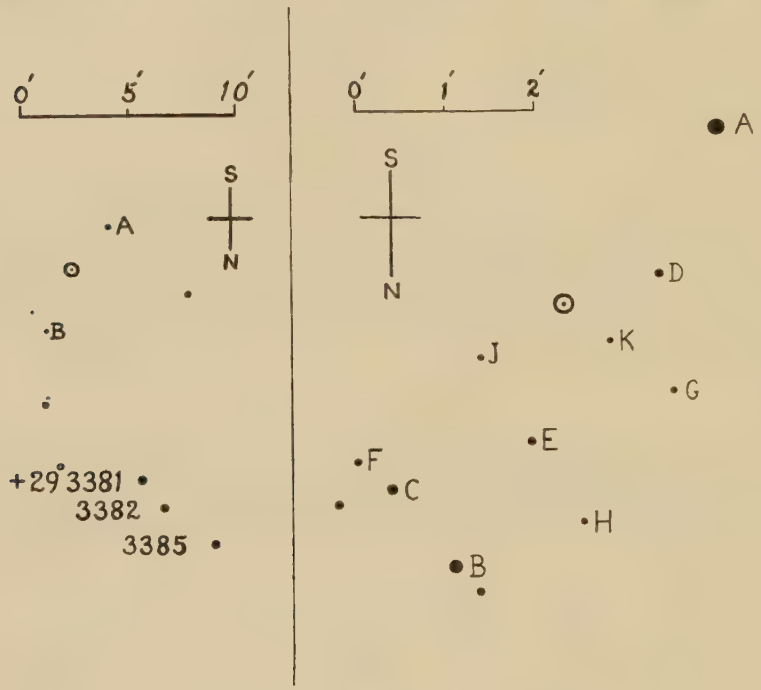

Fig. 2.

for nine years) at a magnitude substantially higher than that from which it rose ; for photographs taken of the region before the outburst, showing sixteenth magnitude stars, record no trace of the nova.

Fig. 2 shows the position of the nova in relation to the nearest B.D. stars, while on the right is shown the immediate neighbourhood of the nova on a larger scale. My estimates of the magnitudes of the comparison stars are as follows :—

A = 11·8	F = 14·7
B = 12·8	G = 15·0
C = 13·3	H = 15·2
D = 14·0	J = 15·3
E = 14·2	K = 15·6.

Nova Ophiuchi (1848)

Observed on forty-seven nights. Very definite variability was exhibited during the period covered by the observations, the total range being $0^{m}.8$, as compared with $0^{m}.4$ in 1933, when the nova was unusually quiescent. It

is difficult to make really satisfactory observations of this object. Its image does not appear to be truly stellar, and it looks relatively large and prominent on the more transparent nights. I have also a strong suspicion that one of the comparison stars regularly used (Barnard's C) is itself slightly variable. It is not possible at present to assign any definite period to the variations of the nova, but a marked maximum is indicated about the middle of July 1934.

Date	Mag.	Date	Mag.	Date	Mag.
1934 May 5	12·9	1934 July 5	12·4	1934 Aug. 11	13·0
10	12·9	6	12·4	12	12·9
11	12·9	7	12·4	15	13·0
17	13·0	8	12·3	16	12·9
18	12·9	9	12·4	18	12·9
22	12·8	10	12·3	19	12·9
30	12·8	11	12·4	29	12·8
31	12·7	14	12·2	30	12·8
June 9	12·8	16	12·2	31	12·8
13	12·7	26	12·5	Sept. 1	12·8
16	12·9	27	12·4	3	12·7
17	12·9	30	12·6	4	12·7
19	12·9	31	12·7	9	12·7
27	12·9	Aug. 3	12·7	10	12·7
July 3	12·6	4	12·8	Oct. 3	12·8
4	12·6	10	12·8		

Nova Persei (1901)

Observed on one hundred and one nights. The usual marked variability of this nova is still maintained, the range in 1934 amounting to $1^{m}.6$. The fluctuations have been less violent than usual, and the average brightness during the whole period covered by the observations was higher than it has been for many years. A marked maximum occurred about the end of 1934 September, and a less marked minimum near the middle of 1935 January. Oscillations of shorter period are superimposed on this gradual rise and fall.

Date	Mag.	Date	Mag.	Date	Mag.
1934 July 5	12·94	1934 Aug. 10	12·90	1934 Sept. 4	12·37
7	12·84	11	12·87	5	12·32
9	12·74	12	12·43	6	12·32
10	12·84	15	12·69	7	12·32
14	12·84	18	12·69	9	12·32
16	12·94	19	12·69	10	12·12
25	12·84	20	12·87	11	12·22
27	12·90	23	12·79	12	12·32
28	12·84	25	12·89	13	12·53
30	12·74	26	12·79	14	12·43
31	12·79	27	12·69	16	12·43
Aug. 2	12·64	29	12·64	17	12·32
3	12·64	30	12·64	18	12·32
4	12·53	31	12·54	21	12·22
6	12·59	Sept. 1	12·54	22	12·12
9	12·79	3	12·43	24	12·07

Date	Mag.	Date	Mag.	Date	Mag.
1934 Sept. 25	12·32	1934 Oct. 22	12·27	1934 Dec. 12	12·95
26	12·02	23	12·64	23	13·10
27	11·82	24	12·74	27	13·10
28	11·82	25	12·64	29	12·92
Oct. 1	11·92	26	12·84	31	13·25
3	12·02	27	12·79	1935 Jan. 4	13·30
4	12·17	28	12·74	10	13·41
5	12·12	29	12·92	25	13·25
8	12·22	30	12·90	27	12·64
9	12·32	31	13·10	Feb. 3	12·82
11	12·32	Nov. 1	13·00	22	12·79
12	12·32	10	12·79	Mar. 4	12·98
14	12·32	13	12·92	12	12·95
15	12·32	22	13·02	20	13·00
16	12·37	Dec. 5	13·20	22	12·95
18	12·32	7	13·15	25	12·95
19	12·37	9	12·95	Apr. 14	13·00
20	12·37	10	12·95		

PHOTO-ELECTRIC LIGHT-CURVES OF δ CEPHEI AND β LYRÆ.

W. M. Smart, M.A., D.Sc.

(Communicated by the Director, Cambridge Observatory)

1. The light-curves given in this paper are based on observations made in the years 1933, 1934 with the photo-electric photometer mounted on the Sheepshanks coudé equatorial. The cell is a potassium cell, filled with argon, its maximum sensitivity being in the blue region of the spectrum. With the cell charged to 180 volts, it is possible to reach, with satisfactory results, magnitude 5·5 for an A0 star. Our procedure in making, for example, a set of three observations, is to obtain readings in the following order :—

$$c,\quad v,\quad c,\quad v,\quad c,\quad v,\quad c,$$

where v denotes the variable star and c the comparison, which acts photometrically as the standard of reference.

The corrections for atmospheric extinction have been applied according to the methods described by the author in previous papers.* The circular aperture through which the light of the star under observation is admitted to the photo-electric cell corresponds to a diameter of about $\frac{3}{4}'$, and is sufficiently small to exclude the light from the companions of the two variable stars under consideration. Corrections have also been applied to render the times of observation heliocentric throughout. The observations were made by Mr. H. E. Green and the writer.

* W. M. Smart, *M.N.*, **94**, 115 and 839, 1933.

δ Cephei

2. The observational material and the discussions of this star are very extensive.* According to Luizet,† the times of maximum are given by

$$\text{J.D. } 2393659{\cdot}856 + 5^{d}{\cdot}366386\ E,$$

and according to Prager ‡ by

$$\text{J.D. } 2393659{\cdot}873 + 5^{d}{\cdot}366396\ E - 0^{d}{\cdot}84{\cdot}10^{-8}\ E^{2}.$$

Using the second formula, we have the following table :—

E	Maximum J.D. 2427000 +	Mean Period
6250	199·520	
		$5^{d}{\cdot}36629$
6300	467·834	
		$5^{d}{\cdot}36629$
6350	736·149	

The star was observed immediately after maximum on 1933 October 25 at J.D. 2427371·380 and, with $E = 6282$, the time of maximum according to the second formula is found to be J.D. 2427371·241. As we have been unable to observe the star precisely at maximum, the agreement is sufficiently

TABLE I

δ Cephei, Light-curve

Phase	No. of Observations	Δm	Weight	Phase	No. of Observations	Δm	Weight
d		m		d		m	
0·085	2	0·338	2	2·485	4	1·040	2
0·110	5	0·289	4	2·535	3	1·067	2
0·368	3	0·191	3	2·687	8	1·115	6
0·424	3	0·173	2	2·724	8	1·116	5
0·442	4	0·178	3	2·898	2	1·127	2
0·461	3	0·183	2	2·931	2	1·144	2
0·671	3	0·285	2	3·155	3	1·211	2
0·786	5	0·352	3	3·392	4	1·252	3
0·837	3	0·363	3	3·524	3	1·294	2
0·988	2	0·499	2	3·563	10	1·318	7
1·156	3	0·543	3	3·978	2	1·420	2
1·248	6	0·562	4	4·066	4	1·427	3
1·278	2	0·600	1	4·191	2	1·420	2
1·309	4	0·608	2	4·496	7	1·415	7
1·577	3	0·738	2	4·530	4	1·417	4
1·762	6	0·755	6	4·552	3	1·394	2
2·042	4	0·904	3	4·995	2	0·974	2
2·100	4	0·928	4	5·007	2	0·955	2
2·217	3	0·963	2	5·173	2	0·690	2
2·288	6	0·983	6	5·224	4	0·634	4
2·387	6	1·025	5	5·244	2	0·603	2

* For bibliography, see *Geschichte und Literatur des Lichtwechsels*, Bd. II, 364, 1920, and also *Zweite Ausgabe*, Bd. I, 279, 1934 (*Veröff. der Univ. Sternwarte Berlin-Babelsberg*).

† *Annales de l'Univ. de Lyon*, Nouvelle Série, Fasc. 33, 1912.

‡ *Kleinere Veröff. Berlin-Babelsberg*, No. 13, 124, 1933.

satisfactory to lend confidence to the adopted period ($5^{d}\cdot36629$) used in the reduction of the observations.

The comparison star used in the observations was η Pegasi (visual mag. $3^{m}\cdot10$, spectrum G0). Altogether, 161 determinations of the magnitude

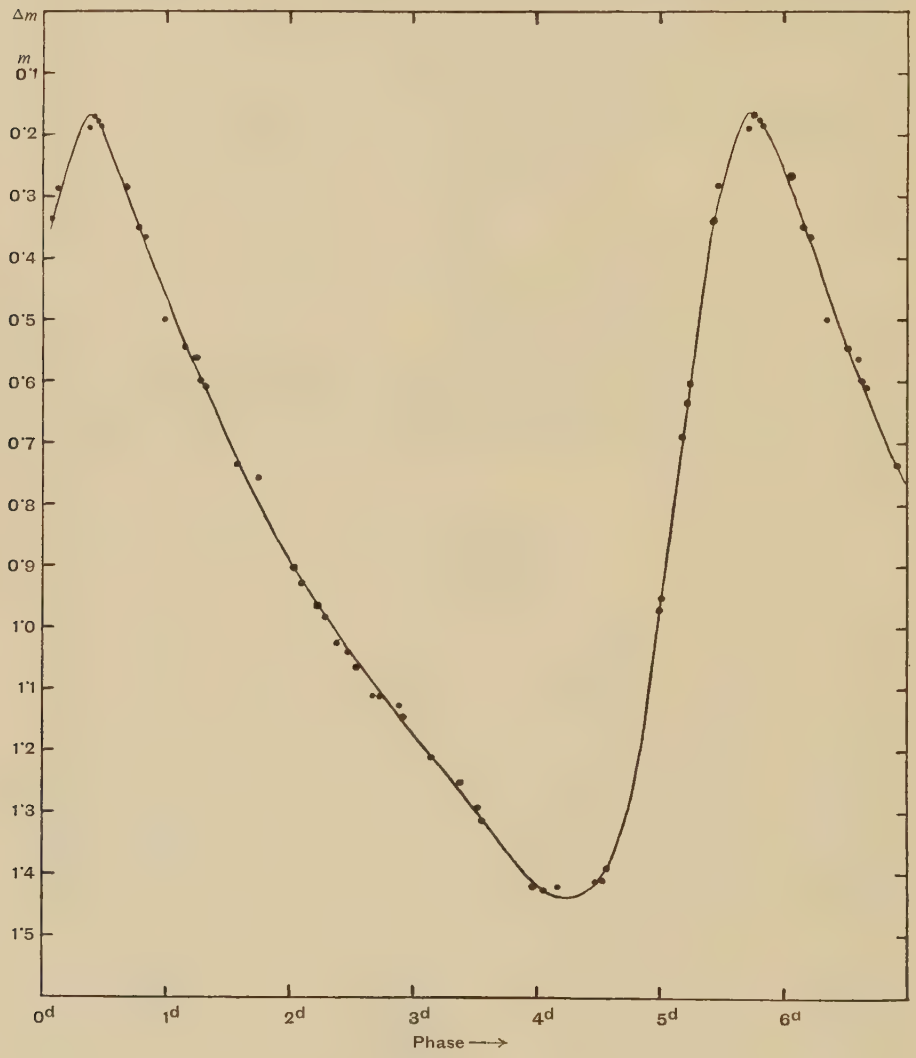

FIG. 1.—*Light-curve of δ Cephei.*

difference $\Delta m \equiv m_{\delta} - m_{\eta}$ were made on 42 nights. In Table I the results of combining the observations are given, the phase being reckoned from J.D. 2427650·000. Fig. 1 shows the light-curve constructed from the data of Table I.

3. According to Prager,* the spectral type of δ Cephei ranges from F4 at maximum to G6 at minimum; according to Shapley,† the range is F1

* *Loc. cit.*

† *Ap. J.*, 44, 290, 1916.

to G2. The comparison star (η Peg.) was selected so as to fall about the mean of the former range, and the extinction factor used in the reductions was based on observations of the two pairs of Go stars, η Pegasi and ι Persei, and η Pegasi and β Draconis.

The range in the visual light of δ Cephei is quoted by Prager as $0^m.72$ and as $0^m.76$ by Pettit and Nicholson.* The photographic range was measured by Wirtz † and found to be $1^m.25$.

Guthnick, using a rubidium cell for which the sensitive wave-length is quoted by Pettit and Nicholson ‡ as about λ 4600, found the photo-electric range to be $1^m.11$. The photo-electric range from the present paper is $1^m.26$, from which it may be inferred that the maximum sensitivity of our potassium cell corresponds closely to that of the photographic region of the spectrum.

β Lyræ

4. In 1933, 206 observations of β Lyræ were made on 41 nights between August 16 and December 24. In 1934 the star was observed on 51 nights and these observations have been divided into two intervals: May 14 to August 5 (123 observations on 21 nights) and August 15 to November 7 (139 observations on 30 nights).

The variability of the light-curve of β Lyræ is well known, and in figs. 2 and 3, which show the principal minimum and the secondary minimum respectively, the deviations from the curve are real and not due to accidental errors as in the light-curve of δ Cephei already considered. According to the late R. H. Curtiss,§ the variation amongst the visual estimates of the maxima and of the principal minima amounts to about $0^m.3$ and in the photo-electric observations || variations of approximately $0^m.2$ have been noted by several observers.

The comparison star used in the present observations is γ Lyræ ($3^m.3$, A_{0p}). It has been suspected by Guthnick and Prager ¶ that γ Lyræ is itself a variable star, but if this is so the much greater fluctuations in the light-curve of β Lyræ cannot be attributed to this source.

The formulæ of Rossiter ** and Prager †† for the times of principal minima have been used in calculating the period, the adopted values of which for the 1933 and 1934 observations are $12^d.9256$ and $12^d.9258$, respectively.

It is possible to obtain a fairly definite estimate of the accuracy of the measures from observations made at or near the times of maxima. As an indication of the probable errors, the following information is given for some

* *Ap. J.*, **78**, 347, 1933.
† *Ast. Nach.*, **154**, 335, 1901.
‡ *Loc. cit.*, p. 346.
§ *Michigan Publ.*, V, 177, 1934.
|| Guthnick, *Sitzungsber.*, 1917, p. 222; Stebbins, *L.O.B.*, **8**, 186, 1916; Huffer, *Washburn Publ.*, XV, Pt. 4, 209, 1931.
¶ *Veröff. Berlin-Babelsberg*, II, Pt. 3, 91 (1918).
** *Michigan Publ.*, V, 67, Equation (2), 1933.
†† *Kleinere Veröff. Berlin-Babelsberg*, No. 11, 1933.

early observations in 1933, any variation in β Lyræ near the time of maxima being ignored.

Maxima	No. of Observations	P.E.	Interval	Extinction Factor, *f*
		m	h	m
1933 Aug. 23	7	± 0·005	2·5	0·39
Sept. 4	8	8	1·8	0·50
,, 5	9	2	1·8	0·44
,, 10	10	7	2·2	0·39

The penultimate column gives the interval during which the observations were made and the last column gives the average value of the extinction factor during the interval concerned.

TABLE II

β Lyræ, Light-curve
1933 *August* 16 *to* 1933 *December* 24

Phase	No. of Observations	Δ*m*	Weight	Phase	No. of Observations	Δ*m*	Weight
d		m		d		m	
0·497	3	0·126	2	7·370	16	0·215	12
0·675	10	0·151	8	7·815	5	0·271	3
0·942	3	0·206	1	8·508	9	0·337	8
1·481	4	0·265	2	9·462	11	0·571	8
1·771	3	0·333	2	9·568	3	0·584	2
2·704	4	0·980	2	9·660	2	0·606	2
2·873	3	1·071	3	10·450	16	0·458	8
3·533	12	1·064	6	10·499	6	0·437	4
4·502	7	0·460	4	10·740	4	0·387	3
4·626	6	0·417	3	11·441	3	0·249	3
4·858	8	0·368	2	11·570	8	0·234	4
5·536	4	0·228	2	11·809	4	0·303	4
5·671	9	0·187	6	12·434	15	0·160	10
5·805	6	0·220	5	12·647	10	0·158	6
6·485	12	0·171	8				

TABLE III

β Lyræ, Light-curve
1934 *May* 14 *to* 1934 *August* 5

Phase	No. of Observations	Δ*m*	Weight	Phase	No. of Observations	Δ*m*	Weight
d		m		d		m	
0·085	7	0·139	6	4·098	4	0·669	3
0·380	8	0·155	8	4·176	8	0·612	6
1·270	1	0·241	1	4·284	4	0·539	4
2·282	5	0·605	5	4·375	4	0·478	4
2·366	6	0·665	6	4·431	2	0·446	1
2·396	12	0·714	10	5·317	4	0·222	2
3·042	2	0·940	2	6·370	5	0·169	4
3·102	6	0·992	6	7·372	5	0·216	5
3·220	8	1·023	7	9·297	3	0·568	1
3·444	10	1·041	9	11·190	4	0·287	2
3·506	9	1·033	9	11·295	6	0·261	4

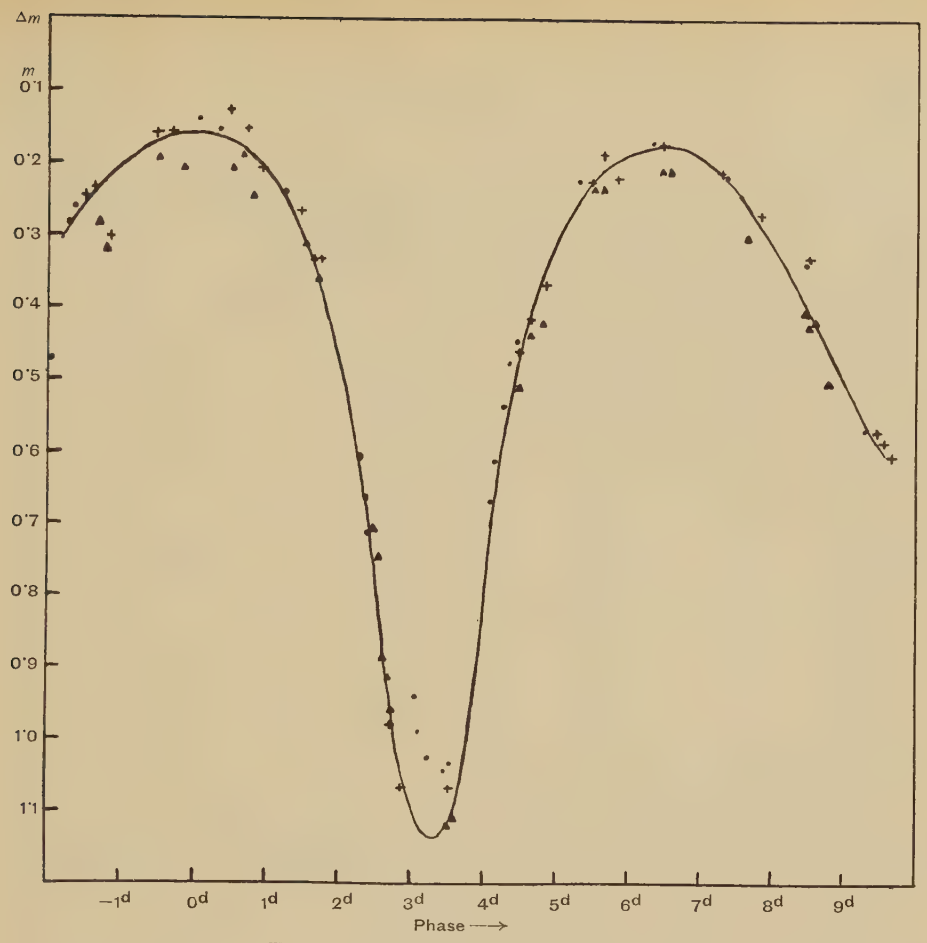

FIG. 2.—*Principal minimum of β Lyræ.*

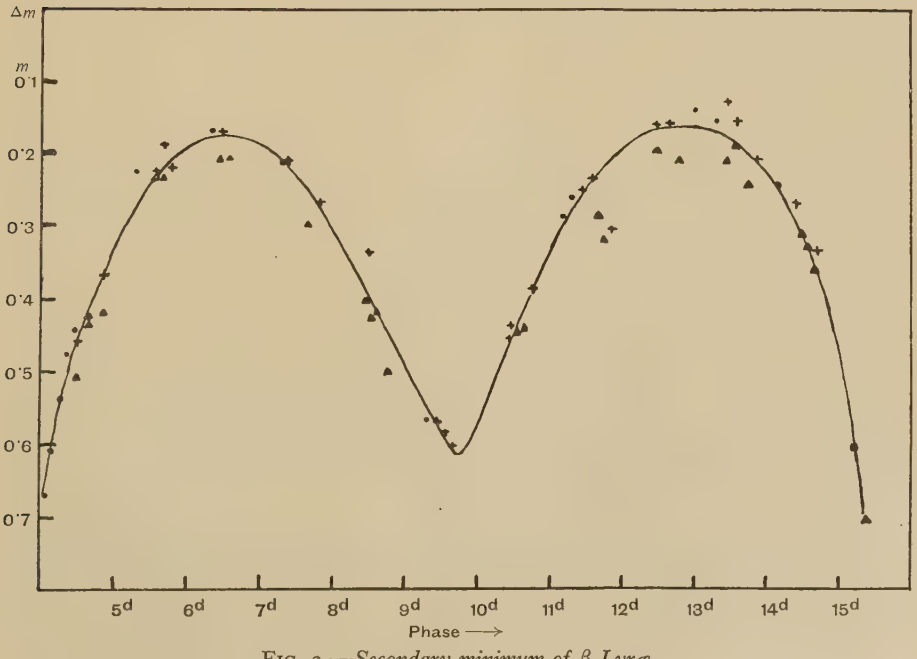

FIG. 3.—*Secondary minimum of β Lyræ.*

\+ *denotes observations* 1933 *Aug.* 16–*Dec.* 24.
● ,, ,, 1934 *May* 14–*Aug.* 5.
Δ ,, ,, 1934 *Aug.* 15–*Nov.* 7.

TABLE IV

β Lyræ, Light-curve
1934 *August* 15 *to* 1934 *November* 7.

Phase	No. of Observations	Δm	Weight	Phase	No. of Observations	Δm	Weight
d		m		d		m	
0·514	4	0·209	3	5·563	6	0·236	3
0·634	3	0·193	3	5·657	7	0·237	5
0·802	3	0·244	3	6·473	6	0·209	6
1·543	5	0·315	3	6·578	6	0·208	4
1·640	4	0·332	4	7·646	4	0·302	3
1·701	3	0·361	3	8·448	3	0·405	2
2·463	3	0·705	3	8·526	3	0·426	3
2·513	3	0·745	3	8·593	8	0·419	6
2·604	4	0·887	4	8·794	2	0·502	1
2·638	4	0·915	4	10·519	5	0·451	5
2·675	2	0·959	2	10·634	5	0·450	4
3·552	6	1·122	5	11·679	6	0·288	3
3·600	7	1·113	6	11·712	4	0·320	2
4·506	6	0·512	4	12·461	4	0·197	4
4·623	4	0·438	4	12·778	4	0·213	3
4·812	5	0·421	4				

Tables II, III and IV give the results of combining the observations during the three intervals concerned, Δm denoting $m_\beta - m_\gamma$. Figs. 2 and 3 show the light-curve for principal and secondary minima respectively. Throughout, the phase is reckoned from

J.D. 2427572·450.

Observatory,
Cambridge :
1935 *June* 8.

NOTE ON TWO ALLIED TYPES OF CHROMOSPHERIC ERUPTIONS.

H. W. Newton.

(Communicated by the Astronomer Royal)

Observations of the Sun's disc in *Hα* light made with the spectrohelioscope at Greenwich since 1930 indicate that eruptive chromospheric phenomena include two types that may be significant in connection with current theories of the solar chromosphere. The recent occurrence of examples of both types within a few days of each other suggested that a note might serve to draw attention to their similarities and points of difference, and to the desirability of securing further data of their very short but energetic life-history.

The phenomena common to both types are the ephemeral occurrence

of brilliant patches or streaks of *Hα* flocculi concurrent with absorption markings possessing high radial velocity.* But in one type (here designated A) the bright emission appears first and is followed immediately or within some minutes by absorption markings (not previously visible) with high radial velocity; in the other case (B) there already exists a definite and sometimes extensive absorption marking of apparent stability, which on the appearance of the brilliant emission develops high radial velocities and dissipates more or less completely within a short time. In both types of eruption the observations suggest that the high radial velocities developed are closely related to the occurrence of the enhanced emission, which itself yields no appreciable radial velocity. So far both types have been observed at Greenwich only near sunspots or where a spot has recently been present.

Type A appears to be considerably more frequent than type B; within the last five years 43 eruptions of the former type have been recorded at Greenwich and only 5 of the latter, though one of these † was extensive and gave rise to a maximum recorded radial velocity of −450 km./sec. It may be noted that the total time spent in observing the Sun's disc is barely 3 per cent. that of the duration of the epoch, 1930–35.

The rapidity with which the phenomena under consideration take place, the sequence of direction of motion and the order of magnitude of the measured radial velocities are illustrated by the values tabulated in Tables I and III for the recent eruptions, viz. 1935 May 6 to illustrate type A and 1935 May 3 and May 10 to illustrate type B. From the extent and brilliance of the patches of *Hα* emission on a provisional scale of 1 (smallest) to 3 (greatest) adopted by Commission No. 11 of the I.A.U., the eruptions of May 3 and 10 will be classed as 2 and that of May 6 as 1.

Eruption of Type A.—1935 May 6^d 9^h U.T. following the rear sunspot of a group in longitude 335°, latitude 30° S., distance from central meridian 28° E.

This eruption was first detected at 8^h 57^m when a small area of abnormally bright *Hα* was visible immediately following the rear sunspot; from this bright patch a fairly narrow but dense absorption marking appeared to extend to a point eastwards about 2′ (or 100,000 km.) as the line-shifter of the spectrohelioscope was turned through readings indicating radial velocities increasing from −60 km./sec., at the point of origin of the dark filament, to −125 km./sec. at its other extremity. A spectroheliogram taken in *Hα* by Mr Evershed at Ewhurst at 7^h 4^m shows a very bright spot just bordering the rear sunspot, but no dark filament had then appeared. The following table shows the progress of the eruption from measurements of the radial velocities of the dark marking or markings first observed at Greenwich at 8^h 57^m. It should be mentioned that the dark absorption changed shape rapidly without changing its general position very appreciably.

* The characteristic phenomena relating to outstanding bright eruptions recorded mainly on spectroheliograms are considered by G. E. Hale in "Solar Eruptions and their Apparent Terrestrial Effects", published in *Ap. J.*, **73**, 379, 1931. See also *M.N.*, **90**, 824, 1930.

† *M.N.*, **91**, 239, 1930.

TABLE I

Time 1935 May 6 U.T.	Measured Radial Velocities	
	Outwards km./sec.	Inwards km./sec.
h m		
8 57 }	−60 to −125	None visible
9 03 }		
9 10	Velocity of −125 has disappeared	
9 11		+29
9 12	−19	
9 16	− 3	+48
9 18	None visible	+38
9 23	,,	+43
9 26	,,	+47
9 27	,,	+57
9 32	,,	+23 to +81 *
9 40	Eruption has disappeared	

The above table shows the initial large outward motions which decrease rapidly with time when inward motions begin to appear; the latter tend to increase before they in turn disappear. It may be added that the small brilliant patch of $H\alpha$ was fading 8 minutes after its first detection and that it possessed no measurable velocity.

Fuller observations bearing on the following points are desirable :—

(1) Is any large outward radial velocity, say $\geqslant$ 50 km./sec., always preceded by or contemporary with a sudden abnormal brightening of $H\alpha$ flocculi?

(2) Are the sequence of radial motions, outwards and then inwards, and the order of velocities as measured in the above case typical of these eruptions?

(3) Is an eruption of this type found only near sunspots?

(4) Observations made at Greenwich show that not every small bright eruption is accompanied by eruptive dark markings (though about 70 per cent. of the observed cases during the last five years have been). Is there any determining factor, such as size or brilliancy of emission, observable?

So far as the evidence is concerned from the series of Greenwich observations, made during the last five years of moderate or slight general activity, it appears that *outward* radial velocities of $\geqslant$ 50 km./sec. seldom occur without some trace of abnormal brightening of $H\alpha$ being noted. Also, that no eruption of type A has been found away from sunspots.

As regards question (2) it may be of interest to give in Table II the following analysis of 23 bright eruptions of type A observed at Greenwich from 1930 to 1935, after discarding those within a short distance of the limb and those very imperfectly observed. With the exception of two, all were of minor extent as regards the area covered by the bright emission.

* Velocity at the penumbra of the sunspot into which one end of the descending structure appeared to be drawn.

Significance of the tabulated values is restricted owing to the fact that these eruptions appear very suddenly, and only in three or four cases was it certain that they had been detected within a few minutes of their commencement. The best observed cases point to the typical sequence of the eruption being similar to that given in Table I, and bearing in mind the observational limitations it may be said that Table II confirms this generality, the tendency for inward velocities to be last seen being definitely shown. The mean respective velocities near the beginning and end of the eruptions are of the order of 50 or 60 km./sec., though this value may be modified when data from a complete solar cycle are available.

TABLE II

Direction and Magnitude of Motion of Eruptive Dark Hα Markings appearing near Strong Hα Emission.

	Outwards	Both Directions	Inwards
Direction when first seen and Mean Radial Velocity	52 per cent. 57 km./sec.	13 per cent. 35 km./sec.	35 per cent. 47 km./sec.
Direction when last seen and Mean Radial Velocity	21 per cent. 51 km./sec.	9 per cent. 35 km./sec.	70 per cent. 52 km./sec.

Eruption of Type B.—(1) 1935 May $3^d\ 8\frac{1}{2}^h$ and (2) May $10^d\ 9\frac{1}{2}^h$. The positions of these were approximately

	Longitude	Latitude	Distance from C.M.
(1)	325°	23° S.	22° E.
(2)	305°	23° N.	50° W.

In both these cases there had been, previous to the eruption, an apparently stable dark marking within an area of *Hα* flocculi of normal brightness that invariably accompany sunspots and outlive them. In the case of (2) the dark marking was very extensive and had been present on the disc for some days previously. In the case of (1) a small group of sunspots was present, but in (2) the sunspot had died out on the day prior to the eruption.

In both instances the unusual activity of the region was detected by the presence of large inward and outward radial velocities belonging to absorption markings of varying shape and intensity, which appeared intermingled and spread over a considerable area when the line-shifter of the spectrohelioscope was oscillated about its normal position, indicating zero radial velocity. At nearly the same time the appearance of streaks of brilliant *Hα* flocculi was noticed, but these had no measurable velocity. In the case of (2) these brilliant streaks occupied a considerable area mainly to the south of the eruptive dark markings and lasted for over $1\frac{1}{2}$ hours. Photometer measures, reduced on a provisional scale, gave the maximum brightness as about 65 per cent. that of the continuous spectrum 4 A. away from the middle of *Hα*.

It seems clear that in both instances an apparently stable dark marking

suddenly disintegrated * contemporaneously with the appearance of brilliant *Hα* emission. Additional confirmation is afforded by *Hα* spectroheliograms taken by Mr. Evershed at Ewhurst shortly before and after each event.

Table III gives the direction and magnitude of the radial velocities of the absorption markings during each eruption.

TABLE III

Radial Velocities measured during Disintegration of Dark Flocculi contemporary with Enhanced Hα Emission

Time U.T.	Measured Radial Velocities	
	Outwards km./sec.	Inwards km./sec.
(1) 1935 May 3		
h m		
8 38	−65	+47
8 45	−40	+72
8 52	None visible	+42
9 07	,,	+32
9 16	Eruption finished	
(2) 1935 May 10		
h m		
9 27 } 9 33 }	−37	+47
9 38	−42	
9 41		+38
9 48	−27 to −79	
9 50		+28 to +62
9 59	−28 to −50	
10 02		+30
10 09	−35	+34
10 24	−31	
10 26		+53
10 53		+56
10 59 } 11 10 }	− 4	+2 to +55
11 10	Eruption dying out	

The brilliance of *Hα* emission on May 10 was noted especially at $9^h\ 44^m$ just before the maximum velocities recorded for the dark absorption.

The observed number of eruptions of type B is so few that any generalization is uncertain, but the observations suggest that in this type there is a general disintegration in more than one direction though the outward velocities may predominate in magnitude and be the first to appear, as in the case described by the Greenwich observers in *M.N.*, **91**, 240, 1930. As in type A, the inward velocities during the recent eruptions were the last visible.

* The occasional sudden disappearance of a stable dark filament recorded by spectroheliograms taken some hours apart has been noted by Evershed and others, but the actual process on the disc does not appear to have been recorded until the spectrohelioscope increased the scope of observation of eruptive phenomena.

Observations of eruptions of type B are very desirable to define the typical course of the outburst and to supply answers to the following points :

(*a*) Is the sudden disappearance of a dark marking always accompanied by a sudden brightening of $H\alpha$?

(*b*) Does such a process occur in regions away from sunspots and areas of $H\alpha$ flocculi of normal brightness?

Eruption (1) of type B described above affords an instance where only a small sunspot was present, and eruption (2) an instance where no spot was present at the time but only the fading remnants of the attendant normal bright flocculi. In view of the evidence presented by J. Bartels* in diagrams which show that there is sometimes (especially near sunspot minimum) a marked lack of correlation in time between the international magnetic character figures and comparative data for sunspots and *normal* regions of bright flocculi, information under heading (*b*) is of peculiar interest.

In conclusion it may be said that the two types of eruptions described in this note are suggestive of the conditions necessary for the emission of high-speed particles from the Sun, as postulated † by E. A. Milne in his theory of the solar chromosphere, but absorption markings on the disc with radial velocities of the order of 1600 km./sec. have yet to be observed.

1935 *May* 25.

ON THE DIRECTION OF STAR-STREAMING IN THE GALAXY.

S. W. Shiveshwarkar, B.A.

1. *Introduction*

J. H. Oort,‡ assuming Lindblad's hypothesis of a galactic rotation, obtains the following formulæ for the radial and transverse linear velocities relative to the Sun:—

$$\left.\begin{aligned} \text{Radial velocity} &= \rho = rA \sin 2\lambda \\ \text{Transverse velocity} &= \sigma = rA \cos 2\lambda + rB \end{aligned}\right\}, \tag{1}$$

where $\lambda \equiv l - l_0$, l and l_0 being the heliocentric galactic longitudes of the star and the centre of the galaxy respectively. Also A and B are given by

$$A = \tfrac{1}{2}\left(\frac{\Theta_0}{\varpi} - \frac{\partial \Theta_0}{\partial \varpi}\right), \qquad B = \tfrac{1}{2}\left(-\frac{\Theta_0}{\varpi} - \frac{\partial \Theta_0}{\partial \varpi}\right), \tag{2}$$

where ϖ is the distance from the galactic centre and Θ_0 the circular velocity of the mean of the stars in the neighbourhood of the Sun. Also r is the heliocentric distance. The force K in the galaxy was assumed to be radial and dependent only on the distance ϖ. Also

$$\Theta_0 = \sqrt{\varpi K}. \tag{3}$$

* *Terrestrial Magnetism*, **37**, 48, 1932.

† *M.N.*, **86**, 472 1926.

‡ *B.A.N.*, **3**, 120, 1927 ; **4**, 159, 1928.

Both Lindblad and Oort contemplated a pure galactic rotation without expansion or contraction. Oort's * mathematical analysis shows that galactic rotation without expansion or contraction will always lead to star-streaming with the line of vertices (or the major-axis of Schwarzschild's ellipsoid) pointing towards the galactic centre. The direction of star-streaming has been determined by several astronomers from observational material of proper motions and radial velocities, and though their determinations do not agree perfectly, they show a marked difference † of nearly 20° between the direction of star-streaming and the direction of the galactic centre as determined by Shapley ‡ assuming it to be the centre of mean position of the system of Globular Clusters. Assuming that such a large difference is neither the result of observational errors nor that of improperly selected data in Shapley's determination of the galactic centre, the only way, therefore, to get out of the difficulty is to make the hypothesis of a systematic expansion or contraction in addition to the rotation. We thus assume that the mean of the stars near the Sun is moving with a velocity Π_0 along the radius from the galactic centre and a velocity Θ_0 at right angles to it. The object of the present paper is to obtain a formula § for ψ, the angle between the direction of star-streaming and the radius-vector from the galactic centre.

2. *Mathematical Investigation*

For our purposes, it will be sufficient to work in two dimensions. Let ϖ, θ be the usual plane polar co-ordinates in the galactic plane with reference to the galactic centre. Let (Π, Θ) be the corresponding velocities. Let the suffix o distinguish quantities associated with the mean of the stars in the neighbourhood of the Sun. Thus the systematic velocities of this are

$$\Pi_0 = \Pi_0(\varpi, \theta), \qquad \Theta_0 = \Theta_0(\varpi, \theta), \tag{4}$$

where (ϖ, θ) are the co-ordinates of this mean.

Let now the number of stars in the element of area $\varpi d\varpi d\theta$ and with velocities lying between $(\Pi, \Pi + d\Pi)$, $(\Theta, \Theta + d\Theta)$ be

$$dN = f(\varpi, \theta, t, \Pi, \Theta)\varpi d\varpi d\theta d\Pi d\Theta. \tag{5}$$

Then in all problems of stellar dynamics, the distribution function f has to satisfy the equation ||

$$\frac{Df}{Dt} = 0. \tag{6}$$

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* *B.A.N.*, **4**, 159, 1928.

† Eddington's determination of the direction of the vertex from Boss's stars is in galactic longitude 347° (*Stellar Movements*, p. 124, 1914).

‡ *Star Clusters*, p. 22, 1930.

§ Kyrill Ogrodnikoff in *Zeits. f. Astr.*, **4**, 190, 1932, writes $\tan \psi = \frac{\Theta_0}{\Pi_0}$, which appears to the author to be erroneous.

|| Jeans, *Problems of Cosmogony and Stellar Dynamics*, **10**, 230, 1919.

We shall assume the force K in the galaxy to be radial and a function of the distance ϖ only, in which case the equations of motion are

$$\left.\begin{aligned} \dot{\Pi} - \Theta^2/\varpi &= -K \\ \dot{\Theta} + \frac{\Pi\Theta}{\varpi} &= 0 \end{aligned}\right\}, \tag{7}$$

and therefore

$$\begin{aligned} \frac{Df}{Dt} &= \frac{\partial f}{\partial t} + \dot{\varpi}\frac{\partial f}{\partial \varpi} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\Pi}\frac{\partial f}{\partial \Pi} + \dot{\Theta}\frac{\partial f}{\partial \Theta} \\ &= \frac{\partial f}{\partial t} + \Pi\frac{\partial f}{\partial \varpi} + \frac{\Theta}{\varpi}\cdot\frac{\partial f}{\partial \theta} + \left(\frac{\Theta^2}{\varpi} - K\right)\frac{\partial f}{\partial \Pi} - \frac{\Pi\Theta}{\varpi}\cdot\frac{\partial f}{\Theta \partial}. \end{aligned}$$

Thus f has to satisfy the equation

$$\frac{\partial f}{\partial t} + \Pi\frac{\partial f}{\partial \varpi} + \frac{\Theta}{\varpi}\cdot\frac{\partial f}{\partial \theta} + \left(\frac{\Theta^2}{\varpi} - K\right)\frac{\partial f}{\partial \Pi} - \frac{\Pi\Theta}{\varpi}\cdot\frac{\partial f}{\partial \Theta} = 0. \tag{8}$$

To explain the phenomenon of star-streaming in the galaxy, we seek a solution of this equation which will conform to Schwarzschild's ellipsoidal distribution. We thus assume

$$f = f_0 \text{ Exp. } [-h^2(\Pi - \Pi_0)^2 - m(\Pi - \Pi_0)(\Theta - \Theta_0) - k^2(\Theta - \Theta_0)^2], \tag{9}$$

where h^2, k^2, m, f_0 are functions of the co-ordinates.

By the theory of conic sections we have

$$\tan 2\psi = m/(k^2 - h^2). \tag{10}$$

It is our aim to express the right-hand side of equation (10) in terms of observed quantities.

Differentiating (9) logarithmically and substituting in (8) we obtain

$$\begin{aligned} &\Pi\left[\frac{1}{f_0}\cdot\frac{\partial f_0}{\partial \varpi} - \frac{\partial h^2}{\partial \varpi}\Pi^2 - \frac{\partial k^2}{\partial \varpi}\Theta^2 - \frac{\partial m}{\partial \varpi}\Pi\Theta + \Pi\frac{\partial}{\partial \varpi}(2h^2\Pi_0 + m\Theta_0) + \Theta\frac{\partial}{\partial \varpi}(2k^2\Theta_0 \right. \\ &\qquad \left. + m\Pi_0) - \frac{\partial}{\partial \varpi}(h^2\Pi_0^2 + k^2\Theta_0^2 + m\Pi_0\Theta_0)\right] \\ &+\frac{\Theta}{\varpi}\left[\frac{1}{f_0}\cdot\frac{\partial f_0}{\partial \theta} - \frac{\partial h^2}{\partial \theta}\Pi^2 - \frac{\partial k^2}{\partial \theta}\Theta^2 - \frac{\partial m}{\partial \theta}\Pi\Theta + \Pi\frac{\partial}{\partial \theta}(2h^2\Pi_0 + m\Theta_0) + \Theta\frac{\partial}{\partial \theta}(2k^2\Theta_0 \right. \\ &\qquad \left. + m\Pi_0) - \frac{\partial}{\partial \theta}(h^2\Pi_0^2 + k^2\Theta_0^2 + m\Pi_0\Theta_0)\right] \\ &+\left(\frac{\Theta^2}{\varpi} - K\right)[-2h^2\Pi - m\Theta + (2h^2\Pi_0 + m\Theta_0)] - \frac{\Pi\Theta}{\varpi}[-2k^2\Theta - m\Pi \\ &\qquad + (2k^2\Theta_0 + m\Pi_0)] = 0. \end{aligned} \tag{11}$$

This equation must be satisfied for all values of Π and Θ. Equating, therefore, the various coefficients, we obtain :

$$\left.\begin{aligned} &\frac{\partial h^2}{\partial \varpi} = 0, \quad \frac{\partial k^2}{\varpi \partial \theta} + \frac{m}{\varpi} = 0, \quad \frac{\partial m}{\partial \varpi} + \frac{\partial h^2}{\varpi \partial \theta} - \frac{m}{\varpi} = 0, \quad \frac{\partial k^2}{\partial \varpi} + \frac{\partial m}{\varpi \partial \theta} + \frac{2h^2 - 2k^2}{\varpi} = 0 \end{aligned}\right\}. \tag{12}$$

$$\left.\begin{aligned}
&\frac{\partial}{\partial\varpi}(2h^2\Pi_0+m\Theta_0)=0\\
&\frac{\partial}{\varpi\partial\theta}(2k^2\Theta_0+m\Pi_0)+\frac{1}{\varpi}(2h^2\Pi_0+m\Theta_0)=0\\
&\frac{\partial}{\partial\varpi}(2k^2\Theta_0+m\Pi_0)+\frac{\partial}{\varpi\partial\theta}(2h^2\Pi_0+m\Theta_0)-\frac{1}{\varpi}(2k^2\Theta_0+m\Pi_0)=0
\end{aligned}\right\}. \qquad (13)$$

$$\left.\begin{aligned}
&\frac{\partial}{\partial\varpi}(\log f_0-h^2\Pi_0{}^2-k^2\Theta_0{}^2-m\Pi_0\Theta_0)+2h^2K=0\\
&\frac{\partial}{\varpi\partial\theta}(\log f_0-h^2\Pi_0{}^2-k^2\Theta_0{}^2-m\Pi_0\Theta_0)+mK=0\\
&-K(2h^2\Pi_0+m\Pi_0)=0
\end{aligned}\right\}. \qquad (14)$$

In addition to the above equations, there will be a gravitational condition connecting f_0 with K, viz. Poisson's equation, if we suppose that our system is self-contained. For instance, if ν denote the density so that

$$\nu=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}fd\Pi d\Theta=\frac{\pi f_0}{\sqrt{4h^2k^2-m}}, \qquad (15)$$

we have Poisson's equation

$$\frac{1}{\varpi}\frac{\partial}{\partial\varpi}(\varpi K)=4\pi^2Gf_0/\sqrt{4h^2k^2-m}, \qquad (16)$$

where G is the constant of gravitation.

Now there is a likelihood that the relation between f_0 and K as given by (16) may be inconsistent with their relation as given by the equations (14).* In this connection we might recall to our mind a remark made by Sir Arthur Eddington in a paper † published in 1915. Our system, viz. the system of stars in the galaxy that conform to an ellipsoidal velocity distribution, may not be self-contained. In fact, star-streaming is exhibited only by certain types of stars while the force K in (16) is produced by all the stars in the galaxy. The latter form a kind of controlling system and are supposed to have arranged themselves in such a way as to have a circular symmetry making K a function of ϖ only. It is thus clear that we may leave out equation (16) keeping only (14) to give a relation between f_0 and K. As a matter of fact we shall have no occasion to use this relation as we are only concerned with the ratios $h^2 : k^2 : m$ which enter into (10) the formula for ψ. As there will be occasion to compare our results with observations, we proceed in the next section to obtain formulæ for radial and transverse motions relative to the mean of the stars in the neighbourhood of the Sun.

* The author is indebted to Sir Arthur Eddington for bringing this important point to his notice.

† *M.N.*, **76**, 37, 1915.

3. *Radial and Transverse Motions*

In fig. 1, C is the centre of the galaxy, S_0 the mean of the stars in the immediate neighbourhood of the Sun, S the mean of the stars in the direction S_0S whose motions we intend to find. The angles, distances and velocities

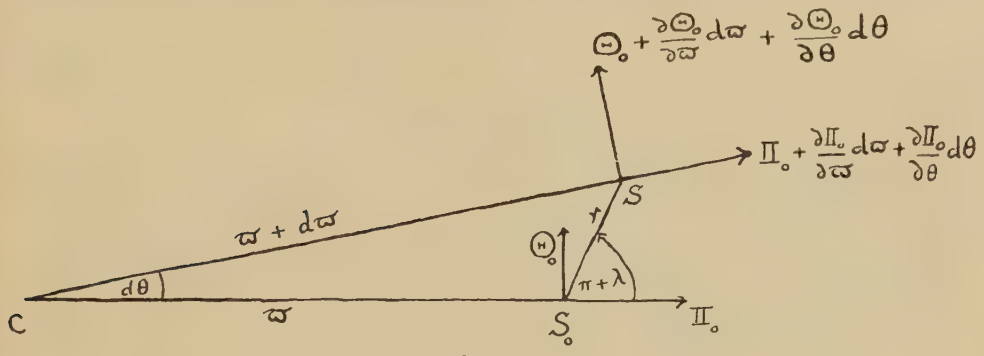

FIG. 1.

are shown in the figure, while the notation has been explained in a previous section. We shall suppose r/ϖ to be small, and neglect $(r/\varpi)^2$ and higher powers.

From the figure

$$\left.\begin{aligned} d\varpi &= r\cos(\pi+\lambda) = -r\cos\lambda \\ \text{and}\quad \varpi d\theta &= r\sin(\pi+\lambda) = -r\sin\lambda \end{aligned}\right\}. \tag{17}$$

Let now ρ and σ denote the radial and transverse motions. The proper motion is σ/r.

Resolving the velocities shown in the figure along and at right angles to S_0S we have

$$\rho = \left(\Pi_0 + \frac{\partial\Pi_0}{\partial\varpi}d\varpi + \frac{\partial\Pi_0}{\partial\theta}d\theta\right)\cos(\pi+\lambda-d\theta)$$
$$+\left(\Theta_0 + \frac{\partial\Theta_0}{\partial\varpi}d\varpi + \frac{\partial\Theta_0}{\partial\theta}d\theta\right)\sin(\pi+\lambda-d\theta) - \Pi_0\cos(\pi+\lambda) - \Theta_0\sin(\pi+\lambda),$$

and

$$\sigma = -\left(\Pi_0 + \frac{\partial\Pi_0}{\partial\varpi}d\varpi + \frac{\partial\Pi_0}{\partial\theta}d\theta\right)\sin(\pi+\lambda-d\theta)$$
$$+\left(\Theta_0 + \frac{\partial\Theta_0}{\partial\varpi}d\varpi + \frac{\partial\Theta_0}{\partial\theta}d\theta\right)\cos(\pi+\lambda-d\theta) + \Pi_0\sin(\pi+\lambda) - \Theta_0\cos(\pi+\lambda).$$

Substituting (17) and simplifying, neglecting terms of second and higher orders of smallness, we get

$$\left.\begin{aligned} \rho &= r(A + B\sin 2\lambda + C\cos 2\lambda) \\ \sigma &= r(A' - C\sin 2\lambda + B\cos 2\lambda) \end{aligned}\right\}, \tag{18}$$

where

$$\left.\begin{aligned} \lambda &= l - l_0 \\ 2A &= \frac{\Pi_0}{\varpi} + \frac{\partial \Pi_0}{\partial \varpi} + \frac{\partial \Theta_0}{\varpi \partial \theta} \\ 2B &= -\frac{\Theta_0}{\varpi} + \frac{\partial \Theta_0}{\partial \varpi} + \frac{\partial \Pi_0}{\varpi \partial \theta} \\ 2C &= -\Pi_0/\varpi + \frac{\partial \Pi_0}{\partial \varpi} - \frac{\partial \Theta_0}{\varpi \partial \theta} \\ 2A' &= \frac{\Theta_0}{\varpi} + \frac{\partial \Theta_0}{\partial \varpi} - \frac{\partial \Pi_0}{\varpi \partial \theta} \end{aligned}\right\} \tag{19}$$

It is clear from the above formulæ that $A = C = 0$ is essential to a pure galactic rotation while $A' = B = 0$ is essential to a pure galactic expansion. As observations show non-zero values for all these four quantities, our assumption of an expansion in addition to a rotation is justified. It has recently been pointed out by Professor E. A. Milne * that under these circumstances it is impossible to obtain the value of l_0 from observations of radial and transverse motions only. We are, therefore, compelled to obtain l_0 from other considerations. We shall adopt $l_0 = 325°$ according to Shapley's determination from the positions and distribution of the Globular Clusters.

In terms of A, B, C, A', it is easy to write down the equations of motion which are

$$\frac{\partial \Pi_0}{\partial \varpi} \cdot \Pi_0 + \frac{\partial \Pi_0}{\partial \theta} \cdot \frac{\Theta_0}{\varpi} - \frac{\Theta_0^2}{\varpi} + K = 0$$

$$\frac{\partial \Theta_0}{\partial \varpi} \Pi_0 + \Theta_0 \left(\frac{\partial \Theta_0}{\varpi \partial \theta} + \frac{\Pi_0}{\varpi} \right) = 0,$$

which by (19) become

$$\left.\begin{aligned} (A + C)\Pi_0 + (B - A')\Theta_0 + K &= 0 \\ (A' + B)\Pi_0 + (A - C)\Theta_0 &= 0 \end{aligned}\right\}. \tag{20}$$

4. *The Formula for* ψ

Eliminating the six quantities $\frac{\partial h^2}{\partial \varpi}, \frac{\partial h^2}{\partial \theta}, \frac{\partial k^2}{\partial \varpi}, \frac{\partial k^2}{\partial \theta}, \frac{\partial m}{\partial \varpi}, \frac{\partial m}{\partial \theta}$ from the seven equations in (12) and (13) we obtain

$$2k^2 \left[\frac{\partial \Theta_0}{\partial \varpi} + \frac{\Theta_0}{\varpi} + \frac{\Theta_0}{\Pi_0} \cdot \frac{\partial \Theta_0}{\varpi \partial \theta} \right] + 2h^2 \left[-\frac{\Theta_0}{\varpi} + \frac{\partial \Pi_0}{\varpi \partial \theta} + \frac{\Pi_0}{\Theta_0} \cdot \frac{\partial \Pi_0}{\partial \varpi} \right]$$

$$+ m \left[\frac{\Theta_0}{\Pi_0} \cdot \frac{\partial \Pi_0}{\varpi \partial \theta} - \frac{\Theta_0}{\varpi} \cdot \frac{\Theta_0}{\Pi_0} + \frac{\partial \Pi_0}{\partial \varpi} + \frac{\Pi_0}{\varpi} + \frac{\Pi_0}{\Theta_0} \cdot \frac{\partial \Theta_0}{\partial \varpi} + \frac{\partial \Theta_0}{\varpi \partial \theta} \right] = 0, \tag{21}$$

which by (19) and the second equation of (20) reduces to

$$2h^2 \Pi_0 + m \Theta_0 = 0, \tag{22}$$

* *M.N.*, **95**, 560, 1935.

showing consistency with the first equation of (13) and the last equation of (14). Thus our equations are self-consistent.

From (13) and (22) we have

$$\left.\begin{aligned} \frac{\partial}{\partial\theta}(2k^2\Theta_0+m\Pi_0)&=0\\ \frac{\partial}{\partial\varpi}(2k^2\Theta_0+m\Pi_0)-\frac{1}{\varpi}(2k^2\Theta_0+m\Pi_0)&=0 \end{aligned}\right\}, \tag{23}$$

giving immediately

$$2k^2\Theta_0+m\Pi_0=p\varpi, \tag{24}$$

where p is an arbitrary constant at present unknown. We proceed to find p.

Differentiating (24) partially with respect to ϖ,

$$p=2k^2\frac{\partial\Theta_0}{\partial\varpi}+m\frac{\partial\Pi_0}{\partial\varpi}+\Pi_0\frac{\partial m}{\partial\varpi}+2\Theta_0\frac{\partial k^2}{\partial\varpi}. \tag{25}$$

From the first equation of (12) and from the first equation of (17),

$$2h^2\frac{\partial\Pi_0}{\partial\varpi}+\Theta_0\frac{\partial m}{\partial\varpi}+m\frac{\partial\Theta_0}{\partial\varpi}=0. \tag{26}$$

Substituting for $\frac{\partial m}{\partial\varpi}$ in (25) from (26),

$$p=2k^2\frac{\partial\Theta_0}{\partial\varpi}+m\frac{\partial\Pi_0}{\partial\varpi}-\frac{\Pi_0}{\Theta_0}\left[m\frac{\partial\Theta_0}{\partial\varpi}+2h^2\frac{\partial\Pi_0}{\partial\varpi}\right]+2\Theta_0\frac{\partial k^2}{\partial\varpi},$$

which by (19) and the second of (20) becomes

$$p=2k^2(A'+B)+m(A+C)+\frac{A-C}{A'+B}\{m(A'+B)+2h^2(A+C)\}+2\Theta_0\frac{\partial k^2}{\partial\varpi}.$$

But by (20) and (22),

$$\frac{2h^2}{m}=\frac{A'+B}{A-C}, \tag{27}$$

$$\therefore\ p=2k^2(A'+B)+m(3A+C)+2\Theta_0\frac{\partial k^2}{\partial\varpi}. \tag{28}$$

It remains now to evaluate $\frac{\partial k^2}{\partial\varpi}$.

From the last equation in (12)

$$\varpi\frac{\partial k^2}{\partial\varpi}=2k^2-2h^2-\frac{\partial m}{\partial\theta}. \tag{29}$$

From the second equation of (12) and the first equation of (23)

$$2k^2\frac{\partial\Theta_0}{\partial\theta}-2\Theta_0m+\Pi_0\frac{\partial m}{\partial\theta}+m\frac{\partial\Pi_0}{\partial\theta}=0,$$

$$\therefore\ \Pi_0\frac{\partial m}{\partial\theta}=2\Theta_0m-2k^2\frac{\partial\Theta_0}{\partial\theta}-m\frac{\partial\Pi_0}{\partial\theta},$$

which by (19) becomes

$$\Pi_0 \frac{\partial m}{\partial \theta} = 2\Theta_0 m - 2k^2\varpi\left[(A-C) - \frac{\Pi_0}{\varpi}\right] - m\varpi\left[(B-A') + \frac{\Theta_0}{\varpi}\right]. \qquad (30)$$

From (28), (29), (30), eliminating $\frac{\partial k^2}{\partial \varpi}$ and $\frac{\partial m}{\partial \theta}$ and reducing, by means of (20)

$$p = -2k^2(A'+B) + m\left[(3A+C) - \frac{2(B^2-A'^2)}{(A-C)}\right].$$

Substituting this value of p in (24), and rearranging

$$m\left[(3A+C) + \frac{2(A'^2-B^2)}{(A-C)} + \frac{(A-C)}{(A'+B)}\Omega\right] = 2k^2[(A'+B)+\Omega], \qquad (31)$$

where $\Omega = \frac{\Theta_0}{\varpi}$ = angular velocity of systematic rotation.

From (27), (31) and (10), we have finally

$$2\cot 2\psi = \frac{2k^2}{m} - \frac{2h^2}{m} = \frac{(3A+C) + \frac{2(A'^2-B^2)}{(A-C)} + \frac{(A-C)}{(A'+B)}\Omega}{(A'+B)+\Omega} - \frac{A'+B}{A-C}, \qquad (32)$$

which is the final formula for ψ.

5. *Comparison with Observations*

A, B, C, A' being known from observations, equation (32) may be used for a variety of purposes. Thus if ψ is given an adopted value, Ω is obtained. Then, if the circular velocity Θ_0 is known, ϖ can be found, and *vice versa*. Or we may adopt a value of Θ_0 and a value of ϖ and deduce ψ from (32).

There are several determinations of A, B, C, A' obtained by various astronomers, notably v. d. Pahlen and Freundlich and J. H. Oort. These have been recently analysed by K. Ogrodnikoff * and are quoted in Table I.

TABLE I

$\psi = 20°$

A	B	C	A'	Ω
0·0301	0·0153	−0·0051	−0·026	0·0230
·0276	·0178	− ·0005	− ·026	·0228
·0290	·0160	− ·0020	− ·026	·0257
·0298	·0144	·0000	− ·026	·0280
·0279	·0173	·0000	− ·026	·0250
·0290	·0160	·0000	− ·026	·0276

Adopting $\psi = 20°$, the values of Ω given by (32) are written in the last column in terms of $\frac{\text{kms./sec.}}{\text{parsec}}$. The average value of Ω appears to be 0·0253 and agrees with a velocity $\Theta_0 = 280$ kms./sec. at a distance of $\varpi = 11000$ parsecs. The author recently † made a determination of Θ_0 from the radial velocities

* *Zeits. f. Astr.*, **4**, 190, 1932. † *M.N.*, **95**, 555, 1935.

of globular clusters and obtained $\Theta_0 = 283$ kms./sec. with a probable error of about 40 kms./sec. The probable errors in A, B, C are $\pm$ 0·002 whilst that of A' is $\pm$ 0·007. Thus the result appears to be fairly in agreement with observations if we accept 11000 parsecs as the distance of the galactic centre.

Summary

On the assumption of a galactic expansion in addition to a galactic rotation, a formula for ψ, the deviation of the line of vertices of star-streaming from the radius to the galactic centre, is obtained in terms of the coefficients of harmonic terms in the radial and transverse stellar motions and the angular velocity of the systematic rotation, on the basis of the fundamental equation $\frac{Df}{Dt} = 0$ of stellar dynamics. The result is compared with observations.

Sidney Sussex College,
Cambridge :
1935 *June* 4.

STAR-STREAMING AND SPIRAL MOTION IN THE STELLAR SYSTEM.

Bertil Lindblad.

In a recent paper * I have advanced the hypothesis that the Taurus and Ursa Major streams follow asymptotic spiral orbits emanating from a central region of high density in the stellar system. It will be shown here that this hypothesis really seems to explain in a rather remarkable way some of the principal facts regarding star-streaming, including deviations from certain conditions which are at first sight demanded by the rotation theory of the galaxy. It may be stated at once, however, that we stand here entirely on the basis of that theory, and that the arguments which follow rather have the character of a completion of the picture. It is well known that the rotation theory demands a general preferential motion in the direction of the radius vector in the galactic plane. The theory, however, does not explain right away the fact that the third axis of the velocity-ellipsoid, which is directed at right angles to the galactic plane, is found smaller than the axis along the radius vector, though it is true that we can include such a condition in a kind of "semi-stationary" state of the system. The average deviation of the vertex direction from the direction towards the centre, and a certain local surplus of stars around the Sun, indicated for all spectral types,† when we compare with the density of an average

* *M.N.*, 95, 12, 1934 (*Stockholm Medd.*, No. 17).

† *Cf.*, *e.g.*, C. Schalén, *Nova Acta R. Soc. Scient. Upsaliensis* (IV), 8, No. 1, 1932 (*Upsala Medd.*, No. 55).

field at larger distances and take into account a reasonable interstellar absorption, are other facts which to a certain extent mar the picture from the standpoint of the abstract theory.

The existence of a certain class of asymptotic spiral orbits proceeding from a dense inner region can be predicted on the basis of classical mechanics, as has been shown in several earlier papers.* The conditions are illustrated by our schematical pictures in fig. 1 and fig. 2. Let R_0 be our distance from the galactic centre, R_1 and R_2 the distances corresponding to certain arrow points, P_1 and P_2, on the envelope curve E in the I_1, I_2 diagram. R_2 is then the radius at which a strong density-gradient is supposed to set in. That such a gradient will actually exist at some point between us and the centre in Sagittarius, in, say, a distance from us of a couple of kiloparsecs, cannot be strictly proved at present, but is probable on account of the great mass

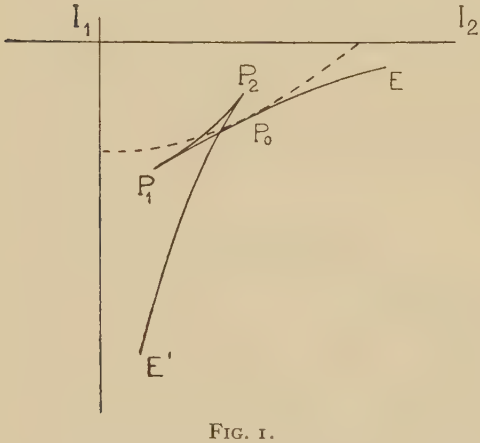

Fig. 1.

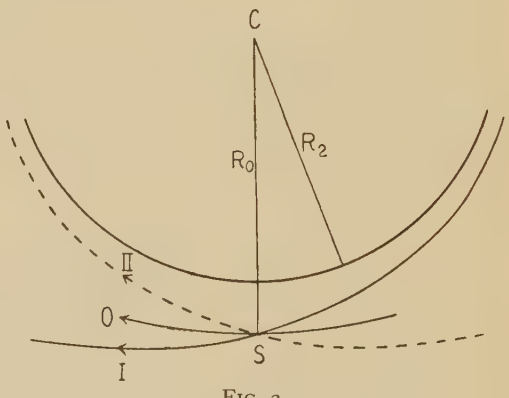

Fig. 2.

of the inner regions, and is rendered possible also by statistical determinations of star-density, when we consider the influence of dark matter.† A few words may be added here to serve as a key to the diagram in fig. 1. The region of the I_1, I_2 plane which has actuality for a certain value of R is the part which lies above a certain parabola $I_2^2 = R^2(I_1 + 2V)$ which touches the envelope curve EE' in a certain point. The parabolæ for successive points along an equatorial radius of the system slide along the envelope, from E' to P_2, from P_2 to P_1 and at last from P_1 to E. A point below P_1P_2 therefore represents the motion of a particle which remains for ever within the radius R_2. In order to escape outside of R_2 the point has to be lifted above P_1P_2. Otherwise the particle in question cannot pass the intermediate distances represented by the parabolæ touching P_1P_2. The appearance of the arrow points P_1 and P_2 underlies the mathematical conditions given in my previous paper referred to above (*cf.* also point (9) below). The combinations of I_1 and I_2 along the curve E represent circular motions at

* *Arkiv för Mat., Astr. och Fysik*, **19**A, No. 35, 1926 (*Upsala Medd.*, No. 13); *K. Svenska Vetenskapsakademiens Handl.*, **4**, No. 7, 1927 (*Upsala Medd.*, No. 31).

† *Cf.* F. H. Seares, *Mount Wilson Contr.*, No. 428 = *Astrophysical Journal*, **74**, 91, 1931.

successive values of R. The coefficient of inclination of the tangent of the envelope at a certain point is equal to twice the angular speed ω_c of the circular motion corresponding to the point in question.

We assume that the matter within R_2 possesses a considerable rate of rotation, and that the representative points of the particles in fig. 1 lie, on the average, closely concentrated towards the envelope branch $E'P_2$, at least in its upper part. Consider now inner irregularities in the distribution of density and resulting slight disturbances of the values of I_1 and I_2 for these particles, contained within the radius R_2 in fig. 2 and below the part P_1P_2 of the envelope in fig. 1. The forces causing these displacements will to a certain extent be analogous to the *viscous forces* of a fluid. Particles near P_2 will then occasionally become elevated over the curve P_1P_2. This means, when we consider fig. 2, that these particles will be sent out from the region within $R=R_2$ in orbits which are nearly asymptotic to the circle of radius R_2. Close to R_2 these orbits have spiral nature, but at larger distances they approach to turning-points, at $R=R_{\max}$, and then return in a symmetrical way towards R_2. As already mentioned, we suppose the displacements ΔI_1 and ΔI_2 in fig. 1 to take place accidentally. In general, therefore, the spiral orbits will be distributed at random with respect to a longitude co-ordinate around the centre C in fig. 2. Each orbit will cut the circle of radius R_0 in two points. The difference of velocity between the spiral orbit and the circular motion of $R=R_0$ will be the same all along the circle in question.

Let us now consider the effect of these motions on the velocity-distribution at a certain point S at the distance R_0 from the centre. This effect can easily be judged from the orbits drawn in fig. 2. We designate the orbits intersecting at S as streams I and II respectively, and identify them with the Taurus and Ursa Major streamings, which we take in their most general signification as forming the nuclei of Kapteyn's two streams, Drift I and Drift II. These streams, when referred to the centroid, occur nearly in the radial direction SC, and their mean velocity coincides nearly with the general motion of stars of low velocity-dispersion. We therefore conclude that the values of I_2 for these streams do not differ much from the value of I_2 for the circular motion at our point of the system, indicated by O in fig. 2. This means, in the language of fig. 1, that the arrow point P_2, which represents closely the state of motion of the streams, lies nearly in the same vertical with P_0, which represents the circular motion at our point of space. As P_0P_2 is small (it is much overdone in the schematical fig. 1), the state of motion represented by P_2 will *everywhere* between R_2 and $R_{\max}$ adhere closely to the circular motions. For greater distances than those corresponding to P_0 the motion will lag slightly behind the circular motion, and for smaller distances to some extent overrun the circular motion. An important thing is that streamings of the kind in question arrive at S with certain *minimum velocity* in relation to the circular motion. Streamings of *higher* relative velocities in roughly the same directions may well occur; they will correspond to motions of higher elevation above P_1P_2 (*cf.* point (7) below).

Some further considerations, which may induce us to favour the present interpretation of the phenomenon of star-streaming, are given in points (1) to (9) below.

1. The predominance of the observed streams in a very wide portion of space around the Sun suggests a very general reason for their appearance, and this demand is actually satisfied by our theory. If the spiral orbits are distributed fairly at random, we must expect the phenomenon to occur in much the same way along the whole circle of radius R_0. Moreover, a phenomenon of the same nature, only slightly different quantitatively, will occur for all values of R between R_2 and R_{max}.

2. It is easy to understand that slight disturbances, especially in the case of the matter passing inwards after having attained R_{max}, may cause the actual vertex direction at S to deviate slightly from the strictly radial direction SC. Further, variations may occur as to the exact position of the representative point of a stream near P_2, above P_1P_2, in fig. 1. A slight difference in this respect between matter outward and inward bound at S may account for the deviation of the vertex direction. The excess of the axis of the velocity ellipsoid in the vertex direction over the third axis becomes an obvious consequence of the theory.

3. A great increase in the central force of galaxy at a distance from us of a couple of kiloparsecs in the direction towards the centre is perhaps indicated by certain peculiarities in the motions of planetary nebulæ and of open clusters in the Sagittarius region. K. F. Bottlinger * has especially pointed out the sudden jump of about 300 km./sec. in the radial velocities of certain planetary nebulæ (objects of absolute amount of radial velocity exceeding 70 km./sec., most of which are nearly "stellar"), when we pass the galactic longitude 330°. A similar though less pronounced effect seems to be present in the case of Miss Hayford's † material of open clusters. These deviations from the regular conditions of differential rotation are perhaps to be put in connection with a considerable and rather sudden increase in the angular speed of rotation, when we approach the interior parts of the system.

4. Near $R=R_{max}$ there will evidently be a general tendency of matter to form a ring separated from the nuclear regions. Therefore, at a point S not far from R_{max}, where fairly massive spiral streams meet, there is likely to appear an apparent "local system," much elongated in a direction perpendicular to the centrum direction SC. If S is the Sun, the direction of elongation coincides nearly with a line joining the constellations Cygnus and Carina. Such a shape of the "local system" has actually been observed.‡ The prominence of our "local system" is, however, no doubt considerably enhanced by the presence in our neighbourhood of the big clouds of B stars, which on the whole follow the circular motion O rather well.

5. The conditions of differential rotation for the matter following nearly the circular motions are in principle entirely the same as in the old theory, because the change of ω_c with R is still continuous. Moreover, within a

* *Veröffentlichungen Berlin-Babelsberg*, X, **2**, 30, 1933.
† *Lick Bulletin*, No. 448, 1932.
‡ *Cf.* B. J. Bok, *Harvard Circular*, No. 371, 1931.

couple of kiloparsecs from the Sun these conditions are likely to be determined by the regular part P_1E of the envelope curve in fig. 1. The interpretation of the phenomenon of the asymmetrical drift as due to a decreasing speed of rotation with increasing velocity-dispersion of the individuals is also entirely unchanged.

6. From a certain point of view we may even include the spiral motions of the streams in the conditions of a quasi-stationary state. We have only to take into account that the streams I and II are likely to be subject to considerable accidental irregularities, and accordingly must expect certain incongruities, for instance in the differential rotation, when material belonging to the streams is included.

7. The stars ejected to some height above P_1P_2 in fig. 1 will appear to us as "high velocity" stars. Now let fig. 3 represent the velocity field in the galactic plane around the Sun. In this figure CO represents the circular

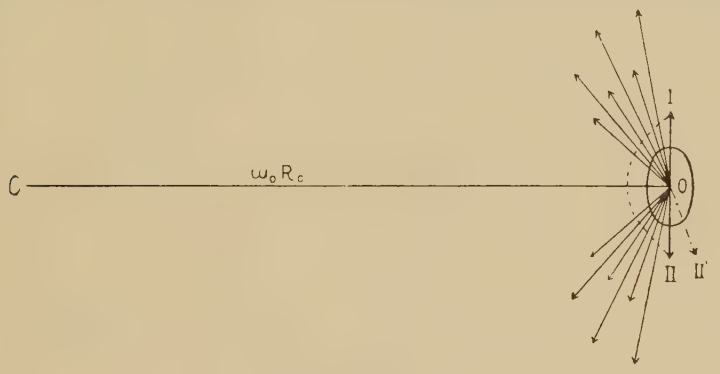

FIG. 3.

velocity $\Theta_0 = \omega_0 R_0$, OI and OII the two star-streams (OII' the actual direction of the Ursa Major stream) and E the "natural" velocity-ellipse corresponding to particles which lie near the envelope branch P_1E in fig. 1. The axes of E, σ_1 and σ_2, thus obey the relation

$$\frac{\sigma_2}{\sigma_1} = \sqrt{1 - \frac{A}{\omega_0}},$$

where A is Oort's coefficient of the differential galactic rotation.

The line P_1P_2 in fig. 1 will now be represented in fig. 3 by the dotted curve combining the points I and II, and the stars ejected above P_1P_2 will be represented schematically by the arrows drawn from the "centroid" O. The observed one-sided distribution of the high velocities is thus very completely accounted for, as well as their preference for the galactic plane.*

8. To what extent our system actually exhibits a spiral structure probably depends on the question, whether certain spiral orbits carry a sufficient amount of matter to impress their form on the appearance of the outer regions of the system. In such a case the visible spiral arms are likely to emerge from antipodal points on the circumference, because the tidal forces

* *Cf.* J. H. Oort, *Groningen Publications*, No. 40, 1926; *B.A.N.*, **4**, 269, 1928.

due to the ejected matter at two opposite points will work in the same direction and most effectively combine in producing a continuation of the process of ejection. We observe, however, that the returning parts of the spiral arms are likely to be much less clearly defined than the outgoing ones. In fact, the matter following the returning parts of the orbits will be much more spread out, because the small accidental velocities of the particles of an arm relative to each other will have had time to produce a considerable effect of scattering. The outgoing branches will therefore appear to dominate almost entirely, and the returning matter will appear much more as an amorphous mass when viewed from a great distance. On the other hand, in the case of nebulæ of very open and massive arms, a certain amount of the outgoing matter probably leaves the system definitely, so that in this case R_{max} lies at infinite distance. A few well-known specimens of spiral nebulæ, however, show a general structure which illustrates very well the conditions

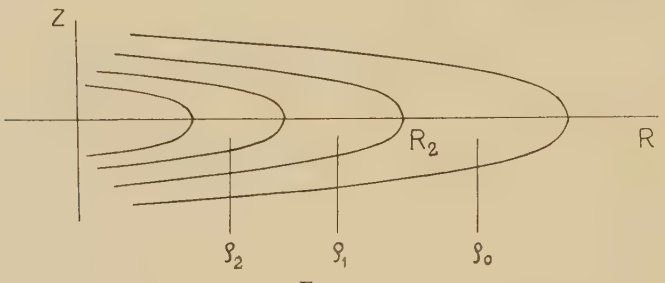

FIG. 4.

demanded in the present theory ; we may mention especially N.G.C. 7217, where the ring at $R = R_{max}$ is very clearly shown.*

9. The existence of the arrow points P_1 and P_2 on E in fig. 1, and thereby the appearance of the spiral orbits, depends on the distribution of density at the distance R_2 from the centre. Let us assume that the density in a certain small interval of R immediately within the distance R_2 from the centre is ρ_1 and for distances somewhat larger than R_2, ρ_0. The equidensity curves are supposed to be roughly spheroidal with a high eccentricity of the meridian section (fig. 4). According to a well-known formula concerning the attraction of spheroids, we have for the component of force vertical to the central plane of an oblate spheroid of density ρ and eccentricity of the meridian section e,

$$\frac{\partial V}{\partial z} = -4\pi G\rho z \frac{1}{e^2}\left[\frac{c}{\sqrt{c^2+x}} - \frac{1}{e}\sqrt{1-e^2}\,\arctan\frac{ce}{\sqrt{(1-e^2)(c^2+x)}}\right]. \qquad (1)$$

c is the small axis of the spheroid, and in the case of an exterior point $c' = \sqrt{c^2+x}$ is the small axis of an ellipsoid confocal with the given spheroid and passing through the point considered. For an interior point $x = 0$. We may now assume the force exerted at a point near the central plane and

* A reproduction of this nebula from a Mount Wilson photograph is given by Jeans in *Astronomy and Cosmogony*, pl. 14, 1928, and by Curtis in *Handbuch der Astrophysik*, V/2, 848, 1933.

just outside of R_2 to be due to a series of spheroids of densities ρ_0, $\rho_1-\rho_0$, $\rho_2-\rho_1$, etc. For the first one of these spheroids $x=0$. For the following ones x may be treated as constant for fixed R and small z, and may further be considered small compared with c^2. We further assume e to approach closely to 1 for the spheroidal surfaces, though we may allow e to increase inwards. We can then write

$$\frac{\partial V}{\partial z}=-4\pi G\bar{\rho}z, \tag{2}$$

where $\bar{\rho}$ is formed as a sum over the positive quantities ρ_0, $\rho_1-\rho_2$, $\rho_2-\rho_1$, etc., after multiplying these quantities with certain coefficients, which for ρ_0 and $(\rho_1-\rho_0)$ approach to 1, but for the later values in the series drop successively. We may call $\bar{\rho}$ the effective density of the vertical component of force. Under the conditions just mentioned it may certainly exceed ρ_0.

We now form the difference ΔI_1 between two points on the envelope curve in fig. 1, one near P_0 and one on P_1P_2 near P_2, assuming $\Delta I_2=0$ between these points. We have

$$\Delta I_1=\Delta\Theta_0{}^2-2\Delta V, \qquad \Theta_0{}^2=-R\frac{\partial V}{\partial R}, \qquad \Delta I_2{}^2=\Delta(R^2\Theta_0{}^2)=0. \tag{3}$$

Developing ΔV we find easily to the second order

$$|\,\Delta I_1\,|=(\Delta R)^2\left(\frac{\partial^2 V}{\partial R^2}+\frac{3}{R}\frac{\partial V}{\partial R}\right)+\cdots, \tag{4}$$

where the derivatives are valid for the point on P_1P_2. This expression suffices for our purpose, because we know that $\frac{\partial^2 V}{\partial R^2}+\frac{3}{R}\frac{\partial V}{\partial R}>0$ is the condition for the appearance of the arrow points P_1 and P_2. Now by Poisson's theorem we have for a point $R=R_2+\delta R$, $z=0$, where δR is a small positive quantity,

$$\frac{\partial^2 V}{\partial R^2}+\frac{1}{R}\frac{\partial V}{\partial R}+\frac{\partial^2 V}{\partial z^2}=-4\pi G\rho_0. \tag{5}$$

We accordingly get by (2), (4) and (5), as $\omega_c{}^2=-\frac{1}{R}\frac{\partial V}{\partial R}$,

$$|\,\Delta I_1\,|=2(\Delta R)^2[2\pi G(\bar{\rho}-\rho_0)-\omega_c{}^2]+\cdots, \tag{6}$$

where, because $\Delta I_2=0$,

$$\omega_c=\frac{R_0{}^2}{R_2{}^2}\omega_0. \tag{7}$$

ω_0 is the angular speed of rotation at our point of the system. The condition derived from (6) will be

$$\bar{\rho}-\rho_0>\frac{\omega_c{}^2}{2\pi G}. \tag{8}$$

Assuming ω_0 to correspond to the linear speed 275 km./sec. for $R_0=10000$ parsec and in (7) $\Delta R=R_0-R_2=2000$ parsec, we get

$$\bar{\rho}-\rho_0>0{\cdot}07 \text{ solar masses per cubic parsec.}$$

The difference between "effective" and actual density just outside of $R = R_2$ will thus have to amount to less than the density at our point of space, which is about 0·1 in the same unit. The condition (8) is therefore very likely to be fulfilled at a point in the interior regions of the system where a fairly strong density-gradient takes place. We may even suppose that such density-gradients occur for several different distances from the centre, causing a series of different systems of spiral orbits to appear, one outside the other. This phenomenon seems actually to occur in some spiral nebulæ, *e.g.* N.G.C. 7217.

We may remark that equation (8) expresses the general theorem that for given ρ_0 and ω_c the appearance of asymptotic spiral orbits only depends on one parameter, the "effective density" $\bar{\rho}$ of the vertical component of force $\frac{\partial V}{\partial z}$. Of course this theorem can also be applied to the case when $\rho_0 = 0$, for instance to the equatorial edge of a homogeneous spheroid. In this case $\bar{\rho} = \rho \, . \, A(e)$, where ρ is the density of the spheroid and $A(e)$ is a function of the eccentricity e of the meridian section. As, however, $\omega_c^2 = \rho \, . \, B(e)$, we get by (8) an equation only containing e, namely $A(e) = \frac{B(e)}{2\pi G}$. The spiral orbits occur in this case for $e > 0{\cdot}834$.

For $e > 0{\cdot}834$ we have $\omega_c^2 < 2\pi G\bar{\rho} < 2\pi G\rho$, as $\bar{\rho} < \rho$. Applying this result to a quasi-spheroidal stellar system, it is remarkable that the condition for the appearance of the spiral orbits becomes so nearly the same as Poincaré's condition for the possibility of internal condensations to form in a "sub-system" of circular speed of rotation, *i.e.* with $\omega = \omega_c$. We may observe that, in general, condensations among the stars can only be expected to form out of stars of nearly identical states of motion, and that in a quasi-spheroidal system we may assume ω_c to be a mean value of ω for any condensation of low internal velocity-dispersion. (In order to avoid misunderstanding we remark that the angular velocity of the homogeneous spheroid is not the same as ω_c, but considerably smaller.) For eccentricities e below 0·834, $\omega_c^2 > 2\pi G\bar{\rho}$, and ω_c^2 is found to fall only slightly below $2\pi G\rho$, so that the average inward force holding a condensation together can only be very slight,* and condensations of low internal velocity dispersion, *i.e.* "local systems" or clusters, are not likely to appear. This state evidently corresponds to the "amorphous" elliptical and spherical nebulæ. At a greater flattening, when the spiral motions at the edge appear, $\frac{\omega_c^2}{2\pi G\rho}$ becomes smaller, and the matter of low internal velocity-dispersion will have a tendency to agglomerate into condensations of various kinds. This conclusion seems actually to be confirmed by the appearance of the very flattened types of extragalactic nebulæ. The tendency to form agglomerations will also favour the aggregation of matter moving in spiral orbits into real massive spiral arms. By the tidal action of the arms on the central mass the process of ejection becomes more intense, enhancing still more the development of the spiral structure (*cf.* point (8) above).

* Cf. *e.g.* Jeans, *Astronomy and Cosmogony*, p. 264, 1928.

The central regions of a spiral nebula which show a fairly low degree of flattening of the equidensity surfaces will remain "amorphous," like the elliptical nebulæ, in the sense that permanent local agglomerations of stars, star clouds or clusters cannot be expected to appear. On the other hand, if we consider a formation of the stars themselves out of an interstellar gas, we must have a condensation process of quite a different nature, where particles of different initial motions go together into one and the same condensation.* In this case we should have to use in Poincaré's theorem the angular velocity ω of the gas, instead of ω_c as above. This condensation process therefore does not fall under the same restrictions as the formation of clusters, and there seems to be no real reason forbidding us to accept the spectrographic evidence that the amorphous nuclear regions of the nebulæ are really composed of stars.

For the system as a whole a proceeding general condensation of gaseous matter into stars, together with an "evaporation" of individuals of high velocity from the system, will cause a general increase of the degree of flattening towards the plane of rotation (*cf.* earlier papers). By the increase of $\frac{\bar{\rho} - \rho_0}{\omega_c^2}$ at certain points in this plane where rapid radial density-gradients occur, the condition (8) for the appearance of spiral orbits (and star-streaming) will become gradually fulfilled. At the same time a disposition for the appearance of local agglomerations of stars in the regions immediately inside these points will increase successively. It seems possible, for instance, that the loose open clusters observed by us in the neighbourhood of the Sun are of fairly recent origin compared with the age of the galactic system itself, and to a considerable extent emanate from those dense regions inside the radius R_2 from which the spiral orbits which are most active in our regions extend.

Stockholm Observatory,
Saltsjöbaden:
1935 *April.*

ON A FORMULA FOR MEAN PARALLAX.

W. J. Luyten.

In a recent issue of the *Monthly Notices* † Smart has published a paper on a simple formula derived by Edmondson ‡ for the mean parallax of stars of known proper motion and radial velocity. While showing Edmondson's derivation of the formula to be erroneous, Smart treats the subject analytically from the point of view of the two-drift hypothesis and comes to the conclusion that the formula itself is valid.

I may be permitted to point out that several years ago I published a series of papers § in which virtually the same analysis as that now given

* Cf. *Nature*, **135**, 133, 1935 (*Stockholm Medd.*, No. 18).
† *M.N.*, **95**, 116, 1934. ‡ *Astr. Journal*, **42**, 22 and 95, 1932.
§ *Proc. Nat. Acad. Sci.*, **11**, 87 and 189, 1925.

by Smart was employed. While my analysis was not so general as that of Smart, I believe it is fair to say that it was more direct. After all, Smart only arrives at *numerical* results by assuming a Maxwellian distribution which I postulated at the beginning. Making the further assumption of random distribution over the sky I established the formula :

$$T_a = T_0\left(1 + \frac{a}{1\cdot 3} - \frac{a^2}{3\cdot 5\cdot 2\cdot} + \frac{a^3}{5\cdot 7\cdot 3\cdot} - \ldots\right),$$

in which T_a is the arithmetic mean value of the *observed* tangential velocity, T_0 the mean "true" tangential velocity corrected for solar motion (*i.e.* Smart's mean peculiar transverse velocity) and $a = h^2V^2$, where h is the characteristic of the Maxwellian distribution and V the drift velocity (*i.e.* solar motion). The reason that my numerical result differs from that of Smart is due to our difference in definition of the "peculiar" velocity, Smart using the residual speed after correcting for stream motion while I corrected only for solar motion.

In deriving the above formula I added : "In a subsequent paper we shall derive a similar expression for the uncorrected radial velocity R_a ; and we shall then be able to make an immediate comparison between crude proper motions and crude radial velocities." * The formula giving R_a in terms of R_0 is found to be identical to the series for T, and hence it is also true that

$$T_a/R_a = \frac{\pi}{2}.\dagger$$

It is easily seen that the same result will hold for any number of drifts provided that their mixture remains constant over the sky.

I do *not* claim to have invented the formula. I merely showed its theoretical validity under certain plausible assumptions—and this as an incidental outcome of a statistical investigation—in 1925, while to the best of my knowledge and belief it had been used as a rough-and-ready indication of parallax for several decades.

As everyone knows who has worked with statistical problems involving proper motions and radial velocities, the formula has its usefulness but also its limitations (*e.g.* it would be unwise to use it, as Edmondson has done, for the first magnitude stars, where the arithmetic mean proper motion has to be derived from some twenty values, when these values themselves range from 3″·7 to 0″·00). It is for the reason of these limitations, and especially in view of the fact that μ and R are measured in different units, and that errors of measurement and distribution trouble their averages in different ways, that the formula is nearly always used in the form that recognizes this basic difference, viz.

$$4{\cdot}74\bar{\mu}/\bar{p} = \frac{\pi}{2}\,.\,\bar{R}.$$

If one must condense all numerical constants into one, then it would seem more logical to the writer to round this off to 3, thus recognizing the

* *Loc. cit.*, p. 90. † *Loc. cit.*, p. 192.

uncertainties involved. The actual error committed is only 0·5 per cent., a negligible quantity in *any* application of the formula, and less than the systematic errors likely to be present in both $\bar{\mu}$ and $\bar{R}$. The formula would then read

$$\bar{p} = 3\bar{\mu}/\bar{R}.$$

The formula has recently been applied by Mohr * to moving clusters, which were chosen because for them very accurate values of the "true" parallaxes can be calculated from the motion of the cluster as a whole. Unfortunately, it is clear from the derivation of the formula that this is perhaps the one case where it may most certainly *not* be applied, since its validity assumes a Maxwellian distribution of peculiar velocities, but also random distribution of the stars with respect to the apex of the Drift motion.

University of Minnesota,
Minneapolis:
1935 *March* 10.

RELATIVISTIC DEGENERACY.

Chr. Møller, D.Phil., and S. Chandrasekhar, Ph.D.

1. It has generally been accepted that the pressure P exerted by a gas is simply the mean rate of transfer of momentum across an ideal surface of unit area in the gas. From this definition it follows that

$$PV = \frac{1}{3}\int_0^\infty N(p) p v_p dp, \tag{1}$$

where $N(p)$ is the number of particles having momenta between p and $p+dp$, and v_p is the velocity associated with the momentum p. For an electron assembly which is completely degenerate ($T=0$) (1) reduces to

$$P = \frac{8\pi}{3h^3}\int_0^{p_0} p^3 \frac{dE}{dp} dp, \tag{2}$$

where E is the kinetic energy of the electron which has a momentum p. If we substitute in (2) the relation between E and p given by relativistic mechanics, we immediately get the relation between PV and N (the total number of electrons in the assembly) for completely degenerate matter.

2. However, Eddington has recently suggested † that (2) is based on a misunderstanding and that we cannot simply insert in (2) the relativistic relation between E and p. We are quite unable to follow his arguments in his

* *Astr. Journal*, 44, 10, 1934.
† *M.N.*, 95, 197, 1935.

first paper, but in his second paper * he has made a more direct derivation of his result, using the energy-stress tensor $T_{\mu\nu}$. Since the use of the energy-stress tensor $T_{\mu\nu}$ in the quantum theory is not new it is now possible to see why Eddington obtains a result different from the usual treatment of a degenerate gas. Eddington defines the energy-stress tensor $T_{\mu\nu}$ as "the expectation value of the differential operator

$$T_{\mu\nu} = -\frac{1}{m}\frac{\partial^2}{\partial x_\mu \partial x_\nu}, \tag{3}$$

where the right-hand side is to be summed in an appropriate (invariant) way." Eddington does not justify his choice of (3) except by the statement "to satisfy tensor conditions." However, in relativistic quantum mechanics one defines the energy-stress tensor in a different way. In fact, the energy-stress tensor for one particle in free space is defined by (*cf.* W. Pauli, *Handbuch der Physik*, 24 (1), 255)

$$T_{\mu\nu} = \frac{c\hbar}{2i}\left(\psi^* \alpha^\nu \frac{\partial\psi}{\partial x_\mu} - \frac{\partial\psi^*}{\partial x_\mu}\alpha^\nu\psi\right), \tag{4}\dagger$$

where α^1, α^2, α^3 are the Dirac matrices and α^4 is here defined as i times the unit matrix; also $x_4 = ict$. In (4) ψ is a solution of the Dirac equation and ψ^* is its conjugate complex.

To go over to the case of N electrons in a finite volume V which satisfy the exclusion principle *one has to consider ψ and ψ^* as non-commuting quantities satisfying the commutation rules established by Jordan and Wigner.*‡

Let $\psi_{\mathfrak{p},s}$ be a suitably normalised eigen-solution satisfying the Dirac equation § and representing a plane wave with a definite value for the momentum $\mathfrak{p}$ and spin s. Since the electrons are confined in a finite volume, $\mathfrak{p}$ takes on only discrete values and, further, s can take two different values corresponding to the two different directions for the spin.

[The discreteness is obtained as usual by imposing the following periodicity condition :—

$$\psi_{\mathfrak{p},s}(x+l_x, y+l_y, z+l_z) = \psi_{\mathfrak{p},s}(x, y, z), \tag{i}$$

where l_x, l_y, l_z define the sides of a rectangular "box" and the volume V is clearly given by

$$V = l_x l_y l_z.$$

From the above condition (i) we find that the eigen values of the components of momentum are given by

$$p_x = \frac{2\pi n_x \hbar}{l_x}; \qquad p_y = \frac{2\pi n_y \hbar}{l_y}; \qquad p_z = \frac{2\pi n_z \hbar}{l_z}, \tag{ii}$$

where n_x, n_y, n_z are arbitrary integers.]

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* We are indebted to Sir A. S. Eddington for allowing us to see a manuscript copy of his paper.

† The tensor $T_{\mu\nu}$ was introduced by Tetrode, *Z. f. Physik*, 49, 858, 1928.

‡ P. Jordan and E. Wigner, *Z. f. Physik*, 47, 631, 1928.

§ For the explicit expressions see C. G. Darwin, *P.R.S.*, 118, 654, 1928.

We now expand ψ and its conjugate complex ψ^* in terms of $\psi_{\mathfrak{p},s}$ and $\psi^*_{\mathfrak{p},s}$ respectively:

$$\psi = \sum_{\mathfrak{p},s} a_{\mathfrak{p},s}\,\psi_{\mathfrak{p},s}, \tag{5}$$

$$\psi^* = \sum_{\mathfrak{p},s} a^*_{\mathfrak{p},s}\,\psi^*_{\mathfrak{p},s}. \tag{6}$$

In (5) and (6) $a_{\mathfrak{p},s}$ and $a^*_{\mathfrak{p},s}$ are q-numbers satisfying the commutability relations (*cf.* Jordan and Wigner, *loc. cit.*).

$$\left.\begin{aligned} a^*_{\mathfrak{p},s}\,a_{\mathfrak{p}',s'} + a_{\mathfrak{p}',s'}\,a^*_{\mathfrak{p},s} &= \delta_{\mathfrak{p},\mathfrak{p}'}\,\delta_{s,s'} \\ a_{\mathfrak{p},s}\,a_{\mathfrak{p}',s'} + a_{\mathfrak{p}',s'}\,a_{\mathfrak{p},s} &= 0 \\ a^*_{\mathfrak{p},s}\,a^*_{\mathfrak{p}',s'} + a^*_{\mathfrak{p}',s}\,a_{\mathfrak{p},s} &= 0 \end{aligned}\right\}. \tag{7}$$

From (4) we now have

$$S = \tfrac{1}{3}(T_{11} + T_{22} + T_{33})$$

$$= \frac{c\hbar}{6i}\sum_{\mathfrak{p},s}\sum_{\mathfrak{p}',s}\psi^*_{\mathfrak{p},s}\left\{a^*_{\mathfrak{p},s}\left(\sum_{\nu=1}^{3}\alpha^\nu\frac{i}{\hbar}p_\nu'\right)a_{\mathfrak{p}',s'} + a^*_{\mathfrak{p},s}\left(\sum_{\nu=1}^{3}\alpha^\nu\frac{i}{\hbar}p_\nu\right)a_{\mathfrak{p}',s'}\right\}\psi_{\mathfrak{p}',s} \tag{8}$$

or

$$S = \frac{c}{6}\sum_{\mathfrak{p},s}\sum_{\mathfrak{p}',s}a^*_{\mathfrak{p},s}\,a_{\mathfrak{p}',s'}\sum_{\nu=1}^{3}(p_\nu + p_\nu')\psi^*_{\mathfrak{p},s}\,\alpha^\nu\psi_{\mathfrak{p}',s}. \tag{9}$$

We shall denote the expectation value of an operator by putting a bar over it. Thus the pressure P is given by the expectation value $\bar{S}$ of S for a state of the whole assembly in which the N lowest states of the particles are occupied. It now follows from the commutation rules that

$$\overline{a^*_{\mathfrak{p},s}\,a_{\mathfrak{p}',s'}} = N_{\mathfrak{p},s}\,\delta_{\mathfrak{p},\mathfrak{p}'}\,\delta_{s,s'}, \tag{10}*$$

where $N_{\mathfrak{p},s} = 1$ if the state $(\mathfrak{p}, s)$ is occupied and zero otherwise. From (9) we now have

$$P = \frac{c}{3}\sum_{\mathfrak{p},s}N_{\mathfrak{p},s}\sum_{\nu=1}^{3}\psi^*_{\mathfrak{p},s}\,\alpha^\nu\psi_{\mathfrak{p},s}p_\nu. \tag{11}$$

Since $c\alpha^\nu$ has the meaning of velocity in Dirac's theory we clearly have

$$c\psi^*_{\mathfrak{p},s}\,\alpha^\nu\psi_{\mathfrak{p},s} = \frac{p_\nu c^2}{EV} = \frac{v_\nu}{V}, \tag{12}$$

where

$$E = c\sqrt{m^2c^2 + \sum_{\nu=1}^{3}p_\nu^2}. \tag{13}$$

Relations (12) and (13) are easily verified using the explicit expressions for $\psi_{\mathfrak{p},s}$ as given by Darwin (*loc. cit.*).

From (11) and (12) we now have

$$PV = \frac{1}{3}\sum_{\mathfrak{p},s}N_{\mathfrak{p},s}\frac{|\,\mathfrak{p}\,|^2c^2}{E} \tag{14}$$

or

$$PV = \frac{1}{3}\sum_{\mathfrak{p},s}N_{\mathfrak{p},s}(\mathfrak{p}, \mathbf{v}), \tag{15}$$

* This is clear from the matrix representations for the $a_{\mathfrak{p},s}$ (*cf.*, for instance, H. Weyl, *Gruppen Theorie und Quanten Mechanik*, pp. 223, 224).

which is precisely our earlier expression (2) and therefore leads to the same final result.

3. Since Eddington prefers to work with standing waves, it may be mentioned here that we could just as well have taken for the set of orthogonal functions the following set of standing waves instead of our $\psi_{\mathbf{p},s}$:—

$$\left.\begin{aligned}\phi_{\mathbf{p},s} &= \frac{1}{\sqrt{2}}(\psi_{\mathbf{p},s}+\psi_{-\mathbf{p},s})\\ \phi_{-\mathbf{p},s} &= \frac{1}{\sqrt{2}}(\psi_{\mathbf{p},s}-\psi_{-\mathbf{p},s})\end{aligned}\right\}. \tag{16}$$

It is easily verified that (12) continues to be true if the $\psi_{\mathbf{p},s}$'s are replaced by the $\phi_{\mathbf{p},s}$'s. Thus (15) continues to be true even if the fundamental set of orthogonal functions were taken to correspond to standing waves.

4. In conclusion we wish to state that we do not intend this note as a reply in any sense to Eddington's papers. We thought it of interest, however, to point out that, starting with the energy-stress tensor as is defined in *relativistic* quantum mechanics and following Eddington's own procedure for calculating the pressure, we are simply led back to the relation between P and N one had earlier derived from (2) by directly inserting in it the relation between E and p given by relativistic mechanics.

Cambridge:
1935 *June* 7.

STELLAR CONFIGURATIONS WITH DEGENERATE CORES. (SECOND PAPER.)

S. Chandrasekhar, Ph.D.

1. In a previous communication * the general problems of stellar structure as they present themselves on the standard model were rediscussed, using the exact relativistic equation of state to describe degenerate matter.† The method developed in I is, however, quite general and consists essentially in relating the completely degenerate gas spheres governed by the differential equation

$$\frac{1}{\eta^2}\frac{d}{d\eta}\left(\eta^2\frac{d\phi}{d\eta}\right) = -\left(\phi^2-\frac{1}{y_0^2}\right)^{3/2}, \tag{1}$$ ‡

* *M.N.*, **95**, 226–260, 1935. This paper will be referred to as I.

† In a recent paper, *M.N.*, **95**, 297, 1935, Eddington has questioned the validity of the relativistic equation of state for degenerate matter which is still generally accepted. There are, however, grounds for not abandoning the accepted form of the equation of state—the arguments are presented in the preceding paper by Dr. Christian Møller and the writer.

‡ This equation was established in the author's paper, *M.N.*, **95**, 207–226, 1935. This paper will be referred to as H.C. II. The earlier paper, *M.N.*, **91**, 456, 1931, will be referred to as H.C. I.

with the wholly gaseous configurations. Since on the standard model approximation for the gaseous configurations the ratio $(1-\beta_1)$ of the radiation pressure to the total pressure is a function of the mass only, the study of the curves of constant mass in the $(R, 1-\beta_1)$-diagram allows a convenient approach to the problem. In this diagram wholly gaseous configurations are represented by lines parallel to the R-axis, while the completely degenerate configurations are represented by points on the R-axis. The relation between these two sets of configurations was obtained by starting with a wholly gaseous configuration of prescribed mass and infinite extension and slowly contracting it and considering whether deviations from perfect gas laws towards degeneracy set in at all and if so when. In this way a *domain of degeneracy* in the $(R, 1-\beta_1)$-diagram was defined in which the configurations must be composite. To fix the precise nature of the curves of constant mass in the domain of degeneracy one requires further assumptions regarding the opacity of the degenerate core, but the problem of relating the degenerate spheres with the gaseous configurations was in principle solved.

2. But the discussion in I was incomplete in so far as the explicit appearance of the physically important parameter, namely, the luminosity L was suppressed by the use of $(1-\beta_1)$ as the main variable. To gain further physical insight it is necessary therefore to transform the discussion of the curves of constant mass in the $(R, 1-\beta_1)$-plane to a discussion of the curves of constant mass in the $(\log L, \log R)$-plane. This is done in Section I of this paper.

3. To complete the discussion we shall have to verify that the general results are not dependent on the very special nature of the model on which they have been obtained. As Jeans has more than once emphasised,* considerable caution is required in interpreting results based on stellar models which make gaseous configurations Emden polytropes of index 3. The more general analysis in which $(1-\beta_1)$ was allowed to vary through the configuration was provided by Jeans.† It follows from his analysis that *for fairly general stellar models the ratio* $(1-\beta_c)$ *of the radiation pressure to the total pressure at the centre of the configuration is a function of the mass only and is independent of the radius.* It is therefore clear that the whole discussion of I (especially that in Section I of that paper) can now be repeated on this more general analysis by considering the curves of constant mass in the $(R, 1-\beta_c)$-plane. In this plane the gaseous configurations are represented by lines parallel to the R-axis, and the relation between these "Emden-Jeans" polytropes to the completely degenerate configurations can be examined as before. This is done in Section II of this paper.

4. Lastly, in Section III various miscellaneous problems which arise are briefly considered.

* *Astronomy and Cosmogony* (Cambridge), chap. iii. Also *M.N.*, **85**, 201, 1925.

† *Astronomy and Cosmogony* (Cambridge), §§ 78–86, 88–92.

Section I.

5. *The Physical Variables.*—As shown in I, equation (55), Eddington's quartic equation can be written in the form

$$M = M_3 \left(\frac{960}{\pi^4} \frac{1-\beta_1}{\beta_1^4} \right)^{1/2}, \tag{2}$$

where

$$M_3 = -4\pi \left(\frac{2A_2}{\pi G} \right)^{3/2} \frac{1}{B^2} \left(\xi^2 \frac{d\theta_3}{d\xi} \right)_1, \tag{3}$$

where A_2, B and the other symbols have the same meaning as in I. M_3 of course represents the upper limit to the mass of a completely degenerate configuration.

If R is the radius of the configuration, then one easily finds that the central density ρ_0 is given by

$$\rho_0 = B \cdot \frac{M}{M_3} \left(\frac{l}{R} \right)^3, \tag{4}$$

where l is the unit of length introduced in I, equation (33), namely,

$$l = \left(\frac{2A_2}{\pi G} \right)^{1/2} \frac{\xi_1(\theta_3)}{B}. \tag{5}$$

Again the central temperature T_0 of the configuration can be determined from the equation

$$T_0 = -\frac{\beta_1 \mu H}{4k} \frac{GM}{R(\xi\theta_3')_1}. \tag{6}*$$

Using (2) and (3), (6) can be rewritten as

$$\frac{4kT_0}{mc^2} = \frac{Ml}{M_3 R} \beta_1. \tag{7}$$

If M^*, R^*, ρ^*, T^* denote the mass, the radius, the density and the temperature when expressed in units of M_3, l, B and $(mc^2/4k)$ respectively then we have

$$M^* = \left(\frac{960}{\pi^4} \frac{1-\beta_1}{\beta_1^4} \right)^{1/2}; \quad \rho_0^* = \frac{M^*}{R^{*3}}; \quad T_0^* = \frac{M^*}{R^*} \beta_1. \tag{8}$$

Also we notice the relation

$$\frac{T_0^{*3}}{\rho_0^*} = \frac{960}{\pi^4} \frac{1-\beta_1}{\beta_1}. \tag{9}$$

6. *Luminosity.*—We start with the equation †

$$L = \frac{4\pi c G M (1-\beta_1)}{\alpha \kappa_c}, \tag{10}$$

* See, for instance, Milne, *Handbuch der Astrophysik*, Band III/1, p. 209.

† A. S. Eddington, *Internal Constitution of the Stars* (Cambridge), p. 124 (equation 90.1).

where κ_c is the central opacity. We shall assume that

$$\kappa = \kappa_1 \frac{\rho^*}{T^{*7/2}}, \tag{11}$$

where κ_1 is the opacity at $\rho = B$ and $T = mc^2/4k$.

From (8), (9), (10) and (11) we derive that

$$L = \frac{4\pi cGM_3}{a\kappa_1}\left(\frac{960}{\pi^4}\frac{1-\beta_1}{\beta_1^4}\right)^{7/4}\beta_1^{7/2}(1-\beta_1)R^{*-1/2}. \tag{12}$$

If we now introduce the unit of luminosity L_1 defined by

$$L_1 = \frac{4\pi cGM_3}{a\kappa_1}, \tag{13}$$

then (12) can be rewritten in the form

$$L^* = (M^*\beta_1)^{7/2}(1-\beta_1)R^{*-1/2}, \tag{14}$$

where L^* is used to denote the luminosity expressed in units of L_1. If we further introduce the quantity $L^*(\beta_1)$ defined by

$$L^*(\beta_1) = \left(\frac{960}{\pi^4}\frac{1-\beta_1}{\beta_1^4}\right)^{7/4}\beta_1^{7/2}(1-\beta_1), \tag{15}$$

then we have from (14) that

$$\log L^* = \log L^*(\beta_1) - \tfrac{1}{2}\log R^*. \tag{16}$$

The first term on the right-hand side of (16) is a function of the mass only, and hence the curves of constant mass in the ($\log L^*$, $\log R^*$) diagram are straight lines.

7. *The Domain of Degeneracy in the* ($\log L^*$, $\log R^*$)*-Diagram.*—In I, § 5, we showed at what stage a configuration of a prescribed mass less than $\mathfrak{M}$ (contracting from infinite extension) would "just begin to develop degeneracy" * at the centre. This occurs when the radius R_0^* of the configuration is given by (I, equation (34))

$$R_0^* = \left(\frac{960}{\pi^4}\frac{1-\beta_1}{\beta_1^4}\right)^{1/6}\frac{1}{x_0(\beta_1)}, \tag{17}$$

where $x_0(\beta_1)$ is such that

$$\frac{f(x_0)}{2x_0^4} = \left(\frac{960}{\pi^4}\frac{1-\beta_1}{\beta_1}\right)^{1/3}. \tag{18}$$

This value of R_0^* substituted in (12) defines the corresponding luminosity L_0^*:

$$L_0^* = \left(\frac{960}{\pi^4}\frac{1-\beta_1}{\beta_1^4}\right)^{5/3}\beta_1^{7/2}(1-\beta_1)(x_0(\beta_1))^{-1/2}. \tag{19}$$

(17) and (19) together define a curve in the ($\log L^*$, $\log R^*$)-plane,

* What is here meant by "degeneracy just beginning to develop" is stated on p. 229 of my last paper (I).

corresponding to the $(R_0^*, 1-\beta_1)$-curve in the $(R^*, 1-\beta_1)$-plane. Thus for any given mass less than $\mathfrak{M}$ the intersection of the line

$$\log L^* = \log L^*(\beta^\dagger) - \tfrac{1}{2}\log R^* \tag{20}$$

with the $(\log L_0^*, \log R_0^*)$-curve defines the stage at which degeneracy would just begin to develop at the centre. In (20), $\beta^\dagger$ represents the value β_1 has for the prescribed mass in the wholly gaseous state. In Table I a set of corresponding pairs of values for $\log L_0^*$ and $\log R_0^*$ is given and the corresponding locus is shown in fig. 1. The part of the plane below this curve defines our *domain of degeneracy* in this plane.

TABLE I

x	$1-\beta_1$	$\log L_0^*$	$\log R_0^*$	x	$1-\beta_1$	$\log L_0^*$	$\log R_0^*$
0	0	$-\infty$	$+\infty$	3·0	·07149	$\bar{2}$·9413	$\bar{1}$·5190
0·2	·00039	$\bar{8}$·2429	0·2982	3·5	·07598	$\bar{1}$·0520	$\bar{1}$·4579
0·4	·00282	$\bar{6}$·6625	·1394	4·0	·07920	$\bar{1}$·1339	$\bar{1}$·4039
0·6	·00793	$\bar{5}$·9546	·0397	4·5	·08158	$\bar{1}$·1969	$\bar{1}$·3556
0·8	·01505	$\bar{4}$·7689	$\bar{1}$·9632	5·0	·08337	$\bar{1}$·2479	$\bar{1}$·3120
1·0	·02305	$\bar{3}$·3221	$\bar{1}$·8995	6·0	·08583	$\bar{1}$·3249	$\bar{1}$·2357
1·2	·03101	$\bar{3}$·7164	$\bar{1}$·8441	7·0	·08739	$\bar{1}$·3817	$\bar{1}$·1706
1·4	·03839	$\bar{2}$·0076	$\bar{1}$·7949	8·0	·08844	$\bar{1}$·4261	$\bar{1}$·1138
1·6	·04496	$\bar{2}$·2290	$\bar{1}$·7503	9·0	·08918	$\bar{1}$·4624	$\bar{1}$·0625
1·8	·05068	$\bar{2}$·4016	$\bar{1}$·7095	10·0	·08972	$\bar{1}$·4931	$\bar{1}$·0182
2·0	·05561	$\bar{2}$·5391	$\bar{1}$·6720	20·0	·09150	$\bar{1}$·6691	$\bar{2}$·7193
2·2	·05983	$\bar{2}$·6506	$\bar{1}$·6372	30·0	·09185	$\bar{1}$·7620	$\bar{2}$·5435
2·4	·06344	$\bar{2}$·7427	$\bar{1}$·6048	∞	·09212	∞	$-\infty$
2·6	·06653	$\bar{2}$·8198	$\bar{1}$·5744				
2·8	·06919	$\bar{2}$·8852	$\bar{1}$·5459				

It is of course clear that the $(\log L_0^*, \log R_0^*)$-curve asymptotically approaches the line

$$\log L^* = \log L^*(\beta_\omega) - \tfrac{1}{2}\log R^* = 1{\cdot}0378 - \tfrac{1}{2}\log R^*, \tag{21}$$

where, as in I, equation (15), β_ω is such that

$$\frac{960}{\pi^4}\,\frac{1-\beta_\omega}{\beta_\omega} = 1. \tag{22}$$

That the $(\log L_0^*, \log R_0^*)$-curve asymptotically approaches the line (21) simply corresponds to the fact that $\mathfrak{M}$ represents the upper limit to the masses for which degeneracy can set in on contraction.

When $M << M_3$ one easily obtains the asymptotic relation (*cf.* I, equation 48)

$$L_0^* \sim \frac{\pi^4}{960}\left(\frac{4}{5}\right)^{33/2} R_0^{*-17}, \tag{23}$$

or

$$\log L_0^* = \bar{3}{\cdot}4073 - 17\log R_0^*, \quad (M^* \to 0). \tag{24}$$

The line (24) is also shown in fig. 1.

8. *The Nature of the Curves of Constant Mass in the Domain of Degeneracy.* —Consider a mass less than M_3. Then for this mass there exists an equili-

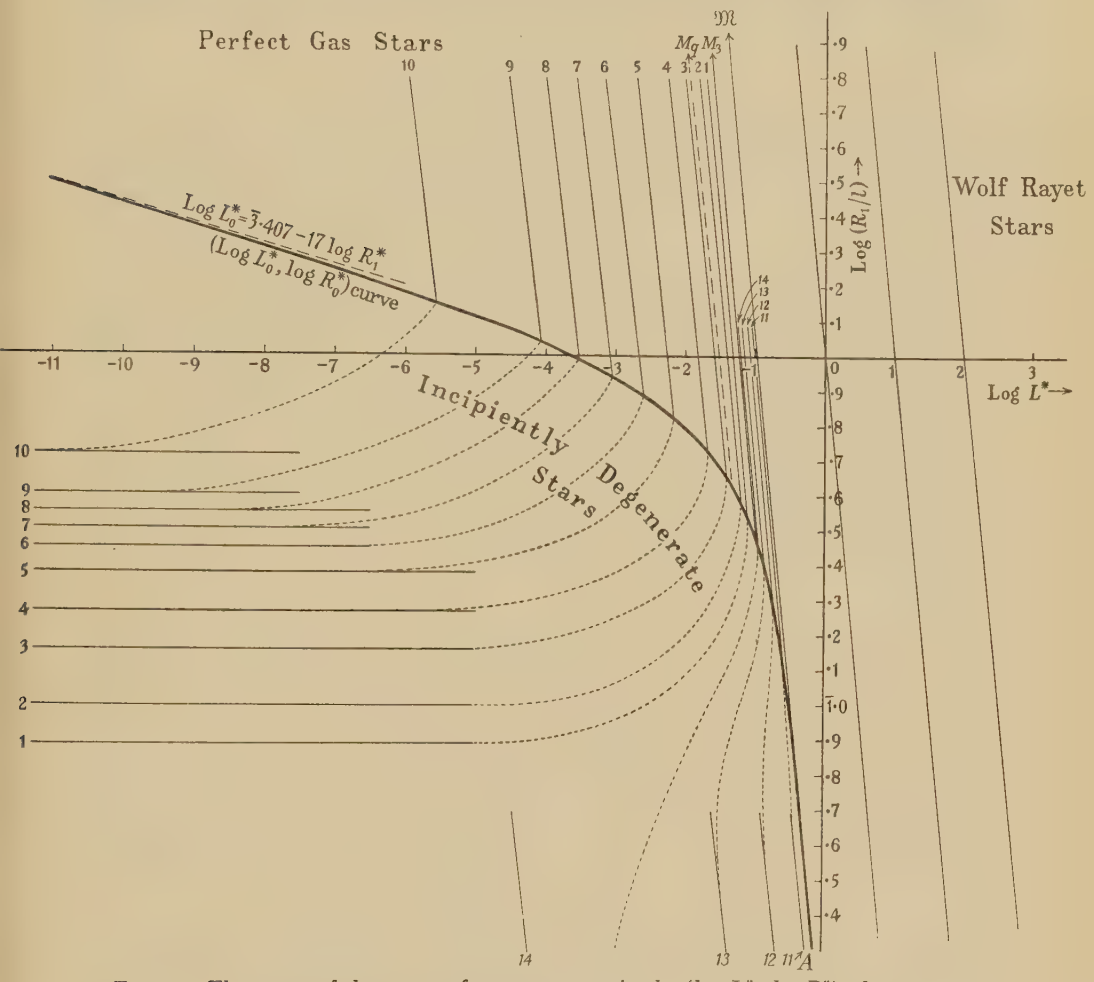

FIG. 1.—*The nature of the curves of constant mass in the (log L*, log R*)-plane on the usual standard model.*

For a general description of the results summarized in the above diagram, see 8 *and also* 23. *On the generalized standard model the system of the curves will look slightly different: thus the continuation of the curves of constant mass marked* (11 . . . 14) *in the perfect gas region will all tend asymptotically to the line* $\mathfrak{M}A$. *But the continuation of the curves of constant mass for those marked* (1 . . . 10) *in the perfect gas region must on all models eventually become asymptotic to the lines* (1 . . . 10) *in the domain of degeneracy (which is the region below the (log* L_0^*, *log* R_0^**)-curve).*

brium configuration in which it is completely degenerate and has a radius R_1^* given by (*cf.* I, equation (46))

$$R_1^* = \frac{1}{y_0(M)} \frac{\eta_1(\phi(y_0(M)))}{\xi_1(\theta_3)}. \tag{25}$$

Hence if we start with this mass and contract it from infinite extension,

then its luminosity increases according to (20) till this line intersects the (log L_0*, log R_0*) locus. On further contraction the configuration develops a degenerate core of finite dimensions and the luminosity must ultimately decrease, and as $L^* \to 0$ the curve must tend asymptotically to

$$\log R^* = \log R_1^* = \text{constant}. \tag{26}$$

In H.C. II we have obtained the values of η_1, M*, etc. for ten different values of y_0, and for these configurations the values of log R_1* and log $L^*(\beta^\dagger)$ can be evaluated. The results of such calculations are given in Table II. The corresponding lines are shown in fig. 1 (the lines marked 1 to 10 in the domain of degeneracy and also in the perfect gas region).

The precise nature of the curves of constant mass in the domain of degeneracy will depend on the assumptions one makes regarding the opacity of the degenerate core. We shall indicate the qualitative results for the two extreme models ($\beta_1 = \beta_2$ and $\beta_2 = 1$) discussed in I, §§ 18–21.

(A) *The Usual Standard Model* ($\beta_1 = \beta_2$).—(i) For $M < M_3$, since the composite configurations are all of the collapsed type, it is clear that as soon as the configuration begins to develop degeneracy at the centre the luminosity should begin to increase less rapidly than $R^{-\frac{1}{2}}$. The curves of constant mass must therefore be of the nature shown by the dotted curves in fig. 1.

TABLE II

$\frac{1}{y_0^2}$	M/M_3	log $L^*(\beta_1)$	log R_1*
0	1·0	$\bar{2}$·7543	$-\infty$
·01	·95733	$\bar{2}$·6670	$\bar{2}$·8903
·02	·92419	$\bar{2}$·5957	$\bar{1}$·0096
·05	·84709	$\bar{2}$·4172	$\bar{1}$·1602
·1	·75243	$\bar{2}$·1690	$\bar{1}$·2708
·2	·61589	$\bar{3}$·7374	$\bar{1}$·3832
·3	·51218	$\bar{3}$·3286	$\bar{1}$·4538
·4	·42600	$\bar{4}$·9116	$\bar{1}$·5095
·5	·35033	$\bar{4}$·4620	$\bar{1}$·5590
·6	·28137	$\bar{5}$·9521	$\bar{1}$·6072
·8	·15316	$\bar{6}$·5173	$\bar{1}$·7198

(ii) The curve for $M = M_3$ bends inwards as it enters the domain of degeneracy and goes to $(-\infty, -\infty)$.

(iii) For $M_3 < M < \mathfrak{M}$ the luminosity initially increases less rapidly than $R^{-1/2}$ on entering the domain of degeneracy, but should again ultimately increase as $R^{-1/2}$, since for these masses the curves of constant mass should be asymptotic to the line

$$\log L^* = \log L^*(\beta^*) - \tfrac{1}{2} \log R^*, \tag{27}$$

where β^* is related to $\beta^\dagger$ (the value β_1 has in the wholly gaseous state) by the equation (*cf.* I, equation (56)),

$$\beta^* = \left(\frac{\pi^4}{960}\frac{\beta^{\dagger 4}}{1-\beta^\dagger}\right)^{1/3}. \tag{28}$$

In fig. 1 this feature of the usual standard model is indicated.

(B) *The Generalized Standard Model* ($\beta_2 = 1$). — (i) For $M \leqslant M_q$ ($\sim 0{\cdot}87 M_3$) the composite configurations are all of the collapsed type and the qualitative nature of the curves of constant mass for these masses must therefore be of the same nature as A (i) above.

(ii) For $M_q < M < M_3$, the composite configurations are initially of the centrally condensed type, and hence for these masses the luminosity will begin to increase *more* rapidly than $R^{-1/2}$ on developing degeneracy at the centre. However, the luminosity must begin to decrease after attaining a certain maximum, since eventually the configurations must tend towards the completely degenerate state.

(iii) For $M_3 \leqslant M < \mathfrak{M}$ the luminosity increases more rapidly than $R^{-1/2}$ in the domain of degeneracy and the curves of constant mass for all these masses must asymptotically tend to the line (*cf.* equations (21), (22))

$$\log L^* = \log L^*(\beta_\omega) - \tfrac{1}{2}\log R^*. \tag{29}$$

9. So far we have restricted ourselves to the standard model approximation, and in relating the completely degenerate configurations with $M \leqslant M_3$ with the wholly gaseous configurations we have seen how such a stellar mass tends to the completely degenerate state when the luminosity tends to zero through a sequence of equilibrium configurations all conforming to the standard model. In addition to M_3 there appeared another mass $\mathfrak{M}$ which played an important rôle in the theory. $\mathfrak{M}$ was initially defined as one for which $\beta^\dagger = \beta_\omega$ (*cf.* I, equations (15), (29)), but the relation between M_3 and $\mathfrak{M}$, namely (I, equation (52)),

$$\mathfrak{M} = M_3 \beta_\omega^{-3/2}, \tag{30}$$

merely means that the existence of an upper limit M_3 to the mass of a completely degenerate configuration and the upper limit $\mathfrak{M}$ to the mass of a configuration for which degeneracy can at all set in on contraction are closely related to one another. Thus configurations in the mass range $M_3 \leqslant M \leqslant \mathfrak{M}$ bridge the gap between masses for which we have equilibrium configurations with zero luminosity and those which cannot develop degenerate cores however far the contraction may proceed.

The existence of a mass $\mathfrak{M}$ is by no means surprising, for, as we have already emphasised in I, § 22, the increased dominance of the radiation pressure for large stellar masses is quite a general result,* and the possibility of degeneracy is entirely excluded if only the radiation pressure is greater than a tenth of the total pressure throughout the entire mass. Hence it is clear that the general features that can be inferred from fig. 1 of this paper,

* An elementary proof of this result is given in the author's report in *Nordisk Astronomisk Tidskrift*, **16**, 37, 1935.

for instance, should to a large extent be independent of the model on the basis of which the discussion has been carried out. It is, however, of interest to verify that this is so by discussing the relation between the completely degenerate configurations and the wholly gaseous configurations on the basis of a more general scheme than what the standard model provides. As the analysis required for this verification is given in Section II below, we shall postpone to Section III some general considerations which arise from a closer examination of fig. 1.

Section II

10. The starting-point of our present discussion is provided by Jeans's investigations on gaseous configurations in which "$1-\beta_1$" is allowed to vary. We shall briefly recapitulate Jeans's analysis in our present notation.

It is clear, of course, that $(1-\beta_1)$ must decrease inwards but the precise law of variation will depend on various factors. If one assumes that

$$\frac{M}{L}\frac{L(r)}{M(r)} \propto T^{\delta}, \tag{31}$$

and that further the coefficient of opacity varies according to the law

$$\kappa \propto \frac{\rho}{T^{7/2}}, \tag{32}$$

then one can easily show that to a fair degree of approximation we have

$$\frac{\beta_1}{(1-\beta_1)^2} = \frac{\beta_c}{(1-\beta_c)}\left(\frac{T}{T_0}\right)^{(\frac{1}{2}-\delta)}, \tag{33}*$$

where β_1 as usual defines the ratio of the gas pressure to the total pressure, and β_c the value of β_1 at the centre of the configuration where the temperature is assumed to be equal to T_0. Equation (33) is valid for layers not immediately near the surface. Further, we have quite generally that

$$\frac{\rho}{\rho_0} = \frac{\beta_1}{1-\beta_1}\frac{1-\beta_c}{\beta_c}\left(\frac{T}{T_0}\right)^3. \tag{34}$$

From (33) and (34) we deduce that

$$\frac{\beta_1}{(1-\beta_1)^2}\left(\frac{\beta_1}{1-\beta_1}\right)^{\frac{1}{3}(\frac{1}{2}-\delta)} = \frac{\beta_c}{(1-\beta_c)^2}\left(\frac{\beta_c}{1-\beta_c}\right)^{\frac{1}{3}(\frac{1}{2}-\delta)}\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}(\frac{1}{2}-\delta)}. \tag{35}$$

From (35) we obtain that

$$[(1+\beta_1)+\tfrac{1}{3}(\tfrac{1}{2}-\delta)]\frac{d\beta_1}{\beta_1(1-\beta_1)} = \tfrac{1}{3}(\tfrac{1}{2}-\delta)\frac{d\rho}{\rho}. \tag{36}$$

Since, however, the total pressure P is given,

$$P = \left[\left(\frac{k}{\mu H}\right)^4 \frac{3}{a}\frac{1-\beta_1}{\beta_1{}^4}\right]^{1/3}\rho^{4/3}, \tag{37}$$

* This equation is due to Jeans and Woltjer.

we have (assuming μ constant *)

$$\frac{dP}{P} = -\frac{1}{3}\frac{4-3\beta_1}{\beta_1(1-\beta_1)}d\beta_1 + \frac{4}{3}\frac{d\rho}{\rho}; \tag{38}$$

or using (36),

$$\frac{dP}{P} = \frac{1}{3}\left[4 - \frac{(\tfrac{1}{2}-\delta)(4-3\beta_1)}{3(1+\beta_1)+(\tfrac{1}{2}-\delta)}\right]\frac{d\rho}{\rho}. \tag{39}$$

On the other hand, if

$$P = K\rho^{1+\frac{1}{n}}, \tag{40}$$

we should have

$$\frac{dP}{P} = \left(1+\frac{1}{n}\right)\frac{d\rho}{\rho}. \tag{41}$$

Comparing (39) and (41) we have for the "*effective polytropic index*" n the expression

$$n = 3 + (1-2\delta)\frac{4-3\beta_1}{1+3\beta_1+2\delta(1-\beta_1)}. \tag{42}$$

Equation (42) shows how the effective polytropic index n varies through the star. (With $\delta = \frac{1}{2}$, $n = 3$ = constant, and we go back to the standard model.) However, Jeans considers that a fair approximation is obtained by regarding the whole configuration as a complete Emden polytrope with an index n_{β_c} given by

$$n_{\beta_c} = 3 + (1-2\delta)\frac{4-3\beta_c}{1+3\beta_c+2\delta(1-\beta_c)}. \tag{43}$$

11. If one assumes (43), then (37) can be rewritten as $(n = n_{\beta_c})$,

$$P = \left[\left(\frac{k}{\mu H}\right)^4 \frac{3}{a}\frac{1-\beta_c}{{\beta_c}^4}\right]^{1/3} \frac{1}{{\rho_0}^{(3-n)/3n}}\rho^{1+\frac{1}{n}}. \tag{44}$$

Or in terms of our A_2 and B we have

$$P = \frac{2A_2}{B^{4/3}}\left(\frac{960}{\pi^4}\frac{1-\beta_c}{{\beta_c}^4}\right)^{1/3} \frac{1}{{\rho_0}^{(3-n)/3n}}\rho^{1+\frac{1}{n}}. \tag{45}$$

The justification for (44) is simply that it gives the same initial variation of P with ρ at the centre as is jointly predicted by (35) and (37) taken together.

With (45) as the "equation of state" the structure of the configuration is completely specified by the Emden function $\theta_{n_{\beta_c}}$ with index n_{β_c}. We easily find that the mass of the configuration is given by

$$M = M_3\left(\frac{960}{\pi^4}\frac{1-\beta_c}{{\beta_c}^4}\right)^{1/2} \mathfrak{J}_M(n_{\beta_c}), \tag{46}$$

where

$$\mathfrak{J}_M(n) = \left(\frac{n+1}{4}\right)^{3/2}\frac{(\xi^2\theta_n')_1}{(\xi^2\theta_3')_1}. \tag{47}$$

* Variation of μ according to the law $\mu \alpha T^j$ can easily be taken into account, but we shall not consider these refinements here.

If $\delta = 1/2$, $n_{\beta_c} = 3$ and $\mathfrak{F}_M(3) = 1$, and (46) reduces to our earlier equation (2). Hence we can rewrite (46) as

$$M_{(\delta)}(\beta_c) = M_{(1/2)}(\beta_c) \cdot \mathfrak{F}_M(n_{\beta_c}), \tag{48}$$

in an obvious notation.

12. *The Domain of Degeneracy in the* $(R, 1-\beta_c)$*-diagram.*—For a given mass M equations (43), (46) and (47) uniquely determine a β_c. In particular there will be a mass for which $(1-\beta_c) = (1-\beta_\omega)$. We shall denote this mass by $\mathfrak{M}_{(\delta)}$. By (46)

$$\mathfrak{M}_{(\delta)} = M_3 \beta_\omega^{-3/2} \mathfrak{F}_M(n_{\beta_\omega}) = \mathfrak{M}_{(1/2)} \mathfrak{F}_M(n_{\beta_\omega}), \tag{49}*$$

where

$$n_{\beta_\omega} = 3 + (1-2\delta)\frac{4-3\beta_\omega}{1+3\beta_\omega+2\delta(1-\beta_\omega)}. \tag{50}$$

Arguing as in I, § 5, we now see that all configurations with $M > \mathfrak{M}_{(\delta)}$ are necessarily wholly gaseous, and that therefore for these masses the curves of constant mass in the $(R, 1-\beta_c)$-diagram are fully represented by the lines parallel to the R-axis. However, for $M < \mathfrak{M}_{(\delta)}$ degeneracy would begin to develop when the central density is given by

$$\rho_0 = B x_0^3, \tag{51}$$

where x_0 is such that

$$\frac{f(x_0)}{2x_0^4} = \left(\frac{960}{\pi^4}\frac{1-\beta_c}{\beta_c}\right)^{1/3}. \tag{52}$$

The radius $R_0(\beta_c\,;\,\delta)$ of this configuration can be determined as in I, § 5, and we find that

$$\frac{R_0(\beta_c\,;\,\delta)}{l} = \left(\frac{n+1}{4}\right)^{1/2}\left(\frac{960}{\pi^4}\frac{1-\beta_c}{\beta_c^4}\right)^{1/6}\frac{1}{x_0}\cdot\frac{\xi_1(\theta_{n_{\beta_c}})}{\xi_1(\theta_3)}. \tag{53}$$

Comparing this with I, equation (34), we deduce that

$$R_0(\beta_c\,;\,\delta) = R_0(\beta_c\,;\,1/2)\mathfrak{F}_R(n_{\beta_c}), \tag{54}$$

where

$$\mathfrak{F}_R(n) = \left(\frac{n+1}{4}\right)^{1/2}\frac{\xi_1(\theta_n)}{\xi_1(\theta_3)}. \tag{55}$$

(53) now defines a curve in the $(R, 1-\beta_c)$-plane. The region enclosed by the two axes and the $(R_0, 1-\beta_c)$-curve now defines our domain of degeneracy in this plane.

13. In our numerical work we shall confine ourselves exclusively to the case $\delta = 0$. Then we have

$$n_{\beta_c} = 3 + \frac{4-3\beta_c}{1+3\beta_c}. \tag{56}$$

The quantities $\mathfrak{F}_R(n)$ and $\mathfrak{F}_M(n)$ are known for certain values of n, and for the intermediate values recourse was made to methods of interpolation.

* It may be noticed here that $\mathfrak{M}_{(1/2)}$ is our original "$\mathfrak{M}$."

14. From (56) we now deduce that

$$n_{\beta_\omega} = 3{\cdot}343\ ; \quad \mathfrak{J}_M(3{\cdot}343) = 1{\cdot}080. \tag{57}$$

Hence

$$\mathfrak{M}_{(0)} = 1{\cdot}080\mathfrak{M}_{(1/2)} = 1{\cdot}249M_3, \tag{58}$$

or putting in numerical values

$$\mathfrak{M}_{(0)} = 7{\cdot}153\mu^{-2}\odot. \tag{59}$$

15. In Table III we have tabulated for this model ($\delta = 0$) the corresponding sets of values for x, $1 - \beta_c$, R_0 and M. The $(R_0,\ 1 - \beta_c)$-curve is shown in fig. 2.

TABLE III

x	$1 - \beta_c$	$n_{\beta c}$	R_0/l	M/M_3
0	0	3·250	∞	0
0·2	·00039	3·250	2·381	·066
0·4	·00282	3·253	1·655	·178
0·6	·00793	3·257	1·320	·301
0·8	·01505	3·264	1·113	·421
1·0	·02305	3·272	·967	·531
1·2	·03101	3·280	·856	·627
1·4	·03839	3·287	·769	·710
1·6	·04496	3·294	·697	·780
1·8	·05068	3·299	·638	·839
2·0	·05561	3·304	·587	·889
2·2	·05983	3·309	·544	·931
2·4	·06344	3·312	·506	·967
2·6	·06653	3·316	·473	·998
2·8	·06919	3·318	·444	1·024
3·0	·07149	3·321	·418	1·046
3·5	·07598	3·326	·365	1·090
4·0	·07920	3·329	·323	1·122
4·5	·08158	3·331	·289	1·145
5·0	·08337	3·333	·262	1·163
6·0	·08583	3·336	·220	1·187
7·0	·08739	3·338	·190	1·202
8·0	·08844	3·339	·167	1·212
9·0	·08918	3·340	·149	1·220
10·0	·08972	3·340	·134	1·225
20·0	·09150	3·342	·067	1·242
30·0	·09185	3·342	·045	1·246
∞	·09212	3·343	0	1·249

16. *The Nature of the Curves of Constant Mass for $M \leqslant M_3$ in the Domain of Degeneracy.*—Let $\beta_c^{\dagger}$ be the value of β_c for a wholly gaseous configuration (of mass less than M_3) which in its completely collapsed state has a central density corresponding to $y = y_0$. Then by equation (46)

$$\frac{\Omega(y_0)}{{\omega_3}^0} = \left(\frac{960}{\pi^4}\,\frac{1 - \beta_c^{\dagger}}{\beta_c^{\dagger 4}}\right)^{1/2} \mathfrak{J}_M(n_{\beta_c}{}^{\dagger}). \tag{60}$$

Now the line through $(1-\beta_c^{\dagger})$ parallel to the R-axis will intersect the $(R_0, 1-\beta_c)$-curve at $(R_0(M(y_0)), 1-\beta_c^{\dagger})$. In the domain of degeneracy the continuation of the curve must in some way connect the point $(R_0(M(y_0)), 1-\beta_c^{\dagger})$ and the point R_1 on the R-axis where

$$\frac{R_1}{l} = \frac{1}{y_0(M)} \frac{\eta_1(\phi(y_0(M)))}{\xi_1(\theta_3)}. \tag{61}$$

In I (Table II) we have already tabulated the values of R_1 for ten different values for y_0 and for these configurations $(1-\beta_c^{\dagger})$ was obtained by interpolating among the figures given in Table III. The corresponding pairs of points on the $(R_0, 1-\beta_c)$-curve and the R-axis are shown in fig. 2. (The points marked 5 to 15 on the R-axis and also on the $(R_0, 1-\beta_c)$-curve.) It is of course clear that for $M = M_3$ the associated curve of constant mass must pass through the origin of our system of axes. If we denote by $\beta_c(0)$ the value of β_c which M_3 has in the wholly gaseous state then we should have by (60) that

$$\left(\frac{960}{\pi^4} \frac{1-\beta_c(0)}{\beta_c^4(0)}\right)^{1/2} \mathfrak{J}_M(n_{\beta_c(0)}) = 1. \tag{62}$$

Numerically $(1-\beta_c(0))$ is found to be 0·0668.

17. *The Nature of the Curves of Constant Mass for $M_3 \leqslant M \leqslant \mathfrak{M}_{(\delta)}$ in the Domain of Degeneracy.*—The discussion of this case will naturally depend on the assumption one makes regarding the opacity in the degenerate core. We shall assume that "$(\kappa\eta)_2$" is constant in the core. Then we have (*cf.* I, equation (63))

$$P = \frac{1}{\beta_2} A_2 f(x) - D_2 \frac{1-\beta_2}{\beta_2}, \tag{63}$$

where D_2 is a constant and

$$\beta_2 = \left(1 - \frac{(\kappa\eta)_2 L}{4\pi c G M}\right). \tag{63'}$$

The reduction to our differential equation (1) for the degenerate core follows at once.

In our present scheme "β" is of course allowed to vary in the gaseous envelope, and we shall in the first instance consider the case where β_2 is just equal to the value of β_1 at the interface between the degenerate core and the gaseous envelope. For this case the discussion can be carried out as in I, § 9.

A completely relativistically degenerate configuration has a mass given by (H.C. I, page 463)

$$M = M_3 \beta^{-3/2}, \tag{64}$$

and is of zero radius. Since we have the further relation

$$\mathfrak{M}_{(1/2)} = M_3 \beta_\omega^{-3/2}, \tag{65}$$

it follows that the curve of constant mass for $\mathfrak{M}_{(1/2)}$ must connect the point $(R_0, 1-\beta_c^{\dagger}(\mathfrak{M}_{(1/2)}))$ on the $(R_0, 1-\beta_c)$-curve and the point $(0, 1-\beta_\omega)$ on the $(1-\beta_c)$-axis. It is therefore clear that for $M_3 \leqslant M \leqslant \mathfrak{M}_{(1/2)}$ the

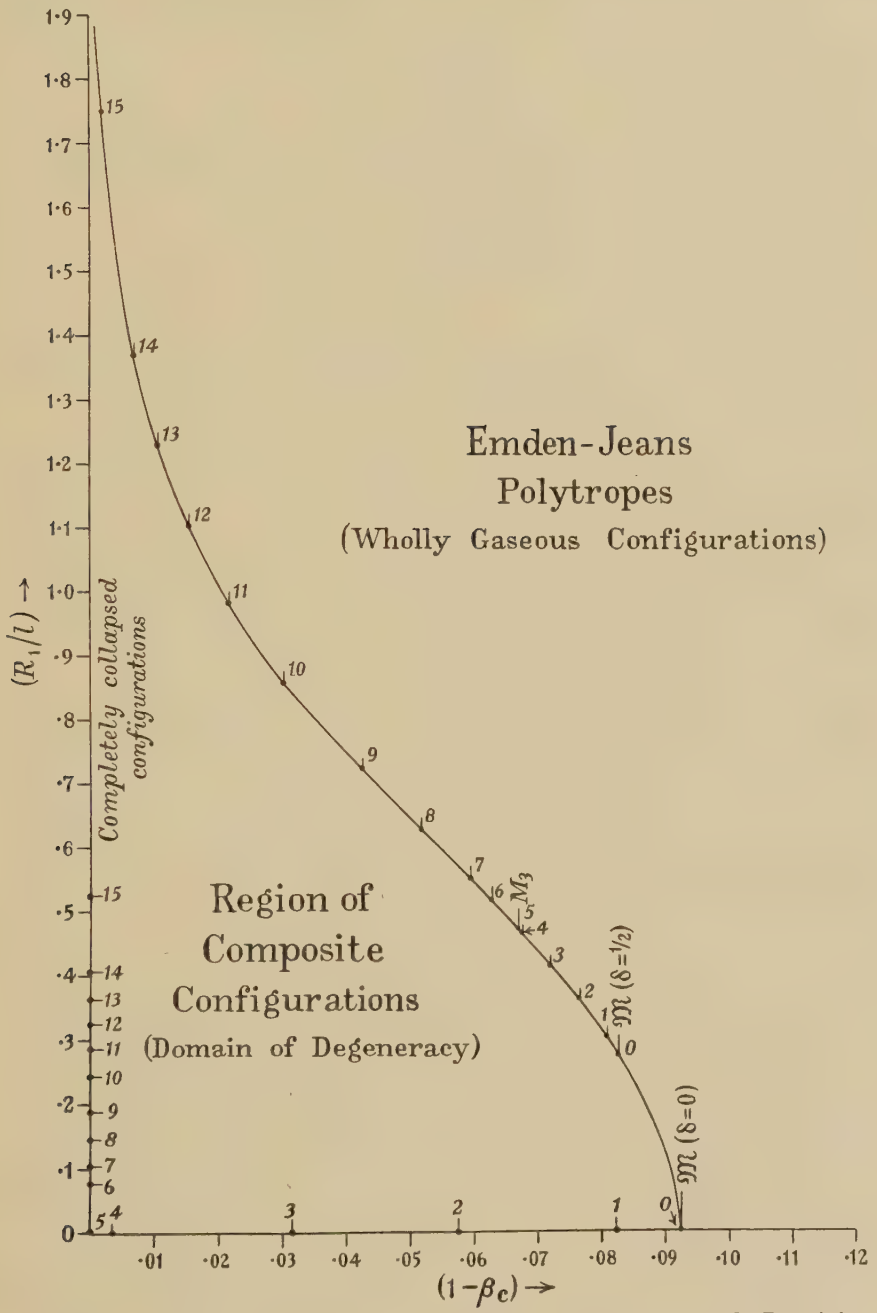

FIG. 2.—*The curve running from* $1-\beta_c=0{\cdot}92\ldots$ *to infinity along the* R_1*-axis is the* $(R_0(\beta_c\,;\ \delta=0),\ 1-\beta_c)$*-curve (see equation* (53)). *The points marked* (5 . . . 15) *on the* $(R_0,\ 1-\beta_c)$*-curve and on the* R_1*-axis are the end points in the domain of degeneracy for the curves of constant mass for the values of* M *for which the* ϕ*-integrals are known (H.C. II). The points marked* (0, 1 . . . 4) *on the* $(R_0,\ 1-\beta_c)$*-curve and on the* $(1-\beta_1)$*-axis are the corresponding end points for some curves of constant mass in the domain of degeneracy on the model discussed in* 17 *(see equation* (67)). *The point* 0 *corresponds to* $\mathfrak{M}_{(\delta=\frac{1}{2})}$, i.e. *our earlier "*$\mathfrak{M}$*."*

curves of constant mass must cross the $(1-\beta_c)$-axis at a point $(1-\beta^*)$ such that

$$M = M_3 \beta^{*-3/2}. \tag{66}$$

If $\beta_c^\dagger$ denotes the value of β_c in the wholly gaseous state then by equating (66) and (46) we have

$$\beta^* = \left(\frac{\pi^4}{960}\frac{\beta_c^{\dagger 4}}{1-\beta_c^\dagger}\right)^{1/3} (\mathfrak{F}_M(n_{\beta_c}{}^\dagger))^{-2/3}. \tag{67}$$

Some corresponding pairs of points on the $(R_0, 1-\beta_c)$-curve and the $(1-\beta_c)$-axis are shown in fig. 2.

Finally, if $\mathfrak{M}_{(1/2)} \leqslant M \leqslant \mathfrak{M}_{(\delta)}$ it is immediately clear that the curves of constant mass consist simply of segments connecting the point $(R_0, 1-\beta_c^\dagger(M))$ on the $(R_0, 1-\beta_c)$-curve to the point $(0, 1-\beta_\omega)$ on the $(1-\beta_c)$-axis.

We thus see that on this model the curves of constant mass in the $(R_0, 1-\beta_c)$-plane combine in the same diagram some of the features of both figs. 3 and 4 of I, obtained on the basis of the two extreme cases of the generalized standard model.

18. It may finally be pointed out that if one assumed that the opacity of the degenerate core is zero then the general qualitative features of the system of the curves of constant mass in the $(R, 1-\beta_c)$-plane must be *exactly* the same as in I, fig. 4.

19. A complete discussion on the basis of Jeans's model will require a study of the composite configurations. The formal theory (which would run similar to I, §§ 11 to 15) can easily be sketched, but as such discussions are not of much interest without the necessary numerical work (which would be considerable) we shall not go into these details here. However, it is clear that the general results derived on the basis of the standard model are fully retained even in this more general analysis.

Section III

20. *The Wolf-Rayet Phenomenon.*—It has already been suggested in I that the Wolf-Rayet phenomenon of the radial ejection of matter may be indirectly due to the fact that the stars of mass greater than $\mathfrak{M}$ (or its equivalent $\mathfrak{M}_{(\delta)}$ on the more general stellar models discussed in Section II) cannot pass *directly* into the white-dwarf stage.* This suggestion is confirmed by observation in so far as general estimates do indicate that Wolf-Rayet stars are massive and dense. On the theoretical side the suggestion gains further support from the following argument:—

Consider a mass greater than $\mathfrak{M}_{(1/2)}$. On the standard model the star must necessarily be wholly gaseous, and we have in a certain system of "natural units" (*cf.* equation (14))

$$L^* = (M^*\beta_1)^{7/2}(1-\beta_1)R^{*-1/2}. \tag{68}$$

* This suggestion (in a rather different form) was independently made to the writer by Dr. W. H. McCrea, to whom the author had earlier communicated a preliminary statement of the main results in the form later published in *Observatory*, 57, 373, 1934.

Further, the value of the surface gravity g is given by

$$g = \frac{GM}{R^2}. \tag{69}$$

From (68) and (69) we have for the ratio X between the integrated flux of radiation πF at the surface of the star to the value of gravity, the expression

$$X = \frac{\pi F}{g} = \frac{L}{4\pi cGM} = \frac{L_1}{4\pi cGM_3} M^{*5/2}\beta_1^{7/2}(1-\beta_1)R^{*-1/2}. \tag{70}$$

From our definition of L_1 in (13) we have

$$X = \left(\frac{1}{a\kappa_1}\right) M^{*5/2}\beta_1^{7/2}(1-\beta_1)R^{*-1/2}. \tag{71}$$

From (71) we see that for a given mass ($> \mathfrak{M}$) the ratio X steadily increases with decreasing R and in fact tends to infinity. This suggests that at some stage in the process of contraction the radiation pressure (a measure of which is given by πF) must overbalance gravity. Ejection of matter must necessarily ensue. In drawing this inference it is of course realized that the deduction from (71) cannot be regarded as a rigorous proof in so far as in our analysis the equations of equilibrium have been integrated up to the boundary. But if one takes this last boundary condition seriously then one cannot also strictly speak of a "mass-luminosity-effective temperature" relation as the temperature has been made zero at the boundary. That this involves no real contradiction was shown in the early writings of Eddington, Jeans and Russell and more recently by Cowling. Bearing this in mind it is now clear that the fact that $X \to \infty$, as $R \to 0$ merely means that the approximations underlying the deduction of the mass-luminosity-radius relation (68) should cease to be valid at some stage. Our conclusion that the ejection of matter must ensue since $X \to \infty$, as $R \to 0$ is now seen to be equivalent to the suggestion that the Wolf-Rayet phenomenon should set in precisely in the region of the Russell diagram where the mass-luminosity-radius relation for the massive wholly gaseous configurations ceases to be valid on the perfect-gas hypothesis itself. It is of course necessary that the star should be massive ($M > \mathfrak{M}$ or its equivalent on more general stellar models) for otherwise we could not extrapolate (68) to high mean densities—degeneracy would have set in earlier for the less massive stars.

21. *The Hydrogen Content of the Massive Stars.*—In § 20 we have used the term "massive stars" to denote those with $M > \mathfrak{M}_{(\delta)}$. It was found on Jeans's model ($\delta = 0$) that we have

$$\mathfrak{M}_{(0)} = 7{\cdot}15\mu^{-2}\odot. \tag{72}$$

To define $\mathfrak{M}_{(\delta)}$ more precisely we need to know the hydrogen content. Depending on the hydrogen content $\mathfrak{M}_{(\delta)}$ can be varied numerically by a factor 16 ($\mu = \frac{1}{2}$ to $\mu = 2$), and it becomes necessary therefore to know at least the minimum hydrogen contents of stars as a function of their mass.

Fortunately we have for our guidance here Strömgren's systematic investigations of this problem.* From Strömgren's work it appears that the molecular weight of the massive B stars already tends towards the lower limit 0·5. Thus we can conclude that for our purposes a star of mass greater than about $25\odot$ can be regarded as "massive."

It is of interest to recall in this connection that in his "Interpretation of the Hertzsprung-Russell diagram" Strömgren says: "With an appreciable overcompressible nucleus the predicted luminosities would be appreciably larger than the observed, and increasing the hydrogen content—as is usually possible to remove the difference—is not possible in these cases, as the limit has already been reached. We conclude then that for the B stars in question there cannot be any appreciable nucleus." We now see that Strömgren's conclusions receive further indirect confirmation from our analysis.

22. *The Hydrogen Content of the White Dwarfs.*—The hydrogen content of the white dwarfs had been investigated earlier by various writers on the Emden polytrope $n = 3/2$ approximation for them. In our H.C. II we have made an exact study of these completely degenerate configurations, and it is now possible to make a more reliable estimate for the appropriate molecular weights for the white dwarfs.

The necessary data required for this calculation are given in Table III of H.C. II. The following table is due to Strömgren :—

TABLE IV

μ for Sirius B

μ	1·91	1·74	1·58	1·44	1·29	0·95
T_{eff} (calculated)	18,300°	13,800°	11,300°	9700°	8300°	6200°

From the above table Strömgren concludes that the value of μ for Sirius *B* should be about 1·6, which means relatively low hydrogen content. The low hydrogen content of the white dwarfs has already been discussed by Strömgren (*loc. cit.*).

23. *Some Remarks on Figure* 1.—Figure 1 is of course the domain of the Hertzsprung-Russell diagram. From an examination of this diagram it is immediately clear that the white dwarfs are placed in their right positions in the Russell diagram. The two essential observational results concerning the white dwarfs, namely, their small mass and low luminosity, receive their natural explanations. The region of the diagram in which we should expect the Wolf-Rayet stars is also indicated. The region of the ordinary stars is indicated by "perfect gas stars." Presumably stars like Krueger 60 are representatives of the "incipiently degenerate" region of our diagram.

The above general conclusions, so far as they go, should clarify the present position regarding stellar structure.

* B. Strömgren, *Z. f. Astrophysik*, **7**, 222, 1933.

24. *Deviations from Perfect Gas Laws arising from Causes other than Degeneracy.*—In our discussion we have so far considered only deviations from perfect gas laws which are due to degeneracy. However, Dirac's theory of the electron predicts a further different type of deviation from the perfect gas laws due to the production of electron pairs at very high temperatures. The bearing of this phenomenon on the theory of stellar structure has been examined in a preliminary communication by L. Rosenfeld and the writer.* As we have indicated in that letter, the deviations from the perfect gas laws arising from this cause are of quite negligible importance for stars with $M < \mathfrak{M}_{(\delta)}$; however, they become increasingly important for the very massive stars. The detailed results of this study will be published separately by Rosenfeld and the writer, but it may be mentioned here that it follows from that study that the production of electron pairs will be of importance in considerations of the structure of stars of masses about 80⊙ and more. The existence of such very massive stars is indicated by the work of J. S. Plaskett, O. Struve, Bottlinger, Trumpler and others, and it seems very probable that the discussion of their structure will lead to some essentially new considerations in the studies on stellar structure.

Finally, it is necessary to point out in this connection that J. von Neumann has recently shown that the *very* ultimate equation of state for matter should *always* be

$$P = \frac{1}{3}c^2\rho. \tag{73}$$

The considerations of this new equation of state does not, however, introduce any essential modifications in our present scheme.

Concluding Remarks.—In two earlier papers (*M.N.*, 95, 207–260, 1935) a first systematic attack was made on the problem of how the conclusions regarding stellar constitution and stellar evolution that have been drawn on the perfect gas hypothesis for the stars have to be modified by the physical possibility of degeneracy in stellar interiors. In this paper the discussion is carried one stage further. Firstly, the physical results have been made more explicit by considering the curves of constant mass in the $(\log L, \log R)$-diagram, which is essentially the domain of the Hertzsprung-Russell diagram. Secondly, the analysis has been extended to include other stellar models more general than the standard model. Thirdly, the bearing of the Wolf-Rayet phenomenon on the evolution of massive stars is examined a little more closely. Certain other miscellaneous questions have also been considered.

In conclusion, I wish to record here my best thanks to Dr. W. H. McCrea, Professor J. von Neumann, Dr. L. Rosenfeld and Dr. B. Strömgren for the encouraging interest they have taken in these studies and for many stimulating discussions.

Trinity College, Cambridge:
1935 *June* 7.

* *Nature*, 135, 999, 1935.

THE ULTRA-VIOLET ABSORPTION SPECTRUM OF NOVA HERCULIS 1934.

H. E. Green, M.A.

(Communicated by the Director, Cambridge Observatory)

At the suggestion of Professor F. J. M. Stratton an objective prism camera, fitted with a 2-inch 30° calcite prism in front of a $2\frac{1}{2}$-inch quartz lens of 18 inches focal length, was mounted on the Thorrowgood telescope, and photographs of the ultra-violet spectrum of Nova Herculis were obtained during the period January 12 to March 8.

The dispersion ranges from 30 A. per mm. at 3100 A. to 60 A. per mm. at 3800 A.

The plate taken on February 26, being in best focus for the extreme ultra-violet, has been completely measured and reduced. For standard lines in the Hartmann-Cornu reduction three easily identified *Ti*II lines, displaced to the violet with a radial velocity of −380 km. per sec., were chosen. The radial velocity was obtained from a slit spectrogram obtained on the same night with the Newall refractor.

In the following table

Column 1 gives the wave-lengths obtained in the reduction.

Column 2 gives the wave-lengths after correction for a displacement of −380 km. per sec. A few lines of *H* and *Ca*II showed a mean displacement of −780 km. per sec. due to the presence of a second absorption component with a velocity of −815 km. per sec., as shown by spectra with a higher dispersion.

Column 3. Quality of line: *vw* = very weak, *w* = weak, *f* = fair, *fg* = fairly good, *g* = good, *vg* = very good, *s* = strong, *br* = broad.

Column 4. Origin.

Column 5. Laboratory wave-lengths.

Column 6. Laboratory intensity.

Column 7. Multiplet nomenclature.

The information in the last four columns has been taken from Miss Charlotte E. Moore's *Multiplet Table of Astrophysical Interest* with the exception of some *Fe*II lines for which details have been supplied by Dr. J. C. Dobbie of the Solar Physics Observatory.

The following lines do not seem to be adequately accounted for by the proposed identification: λ 3264·1; λ 3318·1; λ 3465·1; λ 3581·4.

From the 132 lines measured between λλ 3024–3797, given in the table, 5 lines seem to remain unidentified at present.

The lines in the range for $\lambda < 3140$ were difficult to measure; column 3, Quality, has therefore not been entered. I am obliged to Dr. A. Beer, who has measured this part of the spectrum.

The emission bands are so blended and cut across by absorption lines that

Ultra-violet Spectrum of Nova Herculis 1934.
A—1935 Feb. 26; B—1935 Jan. 12.

H. E. Green, Spectrum of Nova Herculis 1934.

identification proved difficult, and it has not seemed desirable to add a table of those present in the range under examination.

Professor F. J. M. Stratton has supervised all the work and I am grateful for his help.

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3020·5	3024·3		*Ti*II	3022·85	15	$z^2G^\circ_{4\frac{1}{2}} - e^2G_{4\frac{1}{2}}$
			*Ti*II	3023·88	12	$z^2G^\circ_{3\frac{1}{2}} - e^2G_{3\frac{1}{2}}$
3024·7	3028·5		*Ti*II	3029·73	35	$a^2H_{4\frac{1}{2}} - z^2H^\circ_{4\frac{1}{2}}$
3030·2	3034·0		*V*II	3033·43	20	$a^3H_6 - z^3I^\circ_7$
			*V*II	3033·82	30	$a^3G_5 - z^3H^\circ_6$
3032·8	3036·7		*Fe*II	3036·99	5	$z^4P^\circ_{2\frac{1}{2}} - e^4D^\circ_{2\frac{1}{2}}$
3042·6	3046·5		*Ti*II	3046·68	30	$a^4P_{1\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3046·1	3049·0		*V*II	3048·22	20	$a^3H_5 - z^3I^\circ_6$
			*Ti*II	3048·79	6	$b^2D_{2\frac{1}{2}} - y^2P^\circ_{1\frac{1}{2}}$
3049·5	3053·4		*V*II	3053·43	25	$a^3G_4 - z^3H^\circ_5$
3051·5	3055·4		*Ti*II	3056·76	15	$a^4P_{\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$
			*Fe*II	3056·80	5	$a^2I_{6\frac{1}{2}} - x^4G^\circ_{5\frac{1}{2}}$
3054·2	3058·1		*Ti*II	3057·45	10	$a^4F_{1\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Ti*II	3058·08	50	$a^4P_{2\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3061·7	3065·6		*Sc*II	3065·10	5	$z^3F^\circ_4 - e^3G_5$
			*Ti*II	3066·23	30	$a^4F_{2\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Ti*II	3066·36	20	$a^4F_{1\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*V*II	3067·12	25	$a^3G_3 - z^3H^\circ_4$
3067·7	3071·6		*Ti*II	3071·25	15	$a^4P_{2\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$
3070·0	3073·9		*Ti*II	3072·12	30	$a^4F_{3\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
			*Ti*II	3072·98	40	$a^4F_{1\frac{1}{2}} - z^4D^\circ_{\frac{1}{2}}$
3072·2	3076·1		*Ti*II	3075·24	40	$a^4F_{2\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
3073·6	3077·5		*Fe*II	3077·17	10	$a^2I_{6\frac{1}{2}} - z^2H^\circ_{5\frac{1}{2}}$
			*Cr*II	3077·23	2	$a^4F_{4\frac{1}{2}} - z^4H^\circ_{4\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3075·9	3079·8		*Ti*II	3078·66	50	$a^4F_{3\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
3084·6	3088·5		*Ti*II	3088·04	75	$a^4F_{4\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
3086·0	3089·9		*Fe*II	3089·39	4	$c^2D_{2\frac{1}{2}} - x^2G^\circ_{3\frac{1}{2}}$
			*Ti*II	3089·40	15	$b^2G_{3\frac{1}{2}} - x^2F^\circ_{2\frac{1}{2}}$
3089·1	3093·0		*V*II	3093·12	100	$a^5F_5 - z^5G^\circ_6$
3094·5	3098·4		*Ti*II	3097·18	25	$b^4P_{1\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3099·4	3103·3		*V*II	3102·30	80	$a^5F_4 - z^5G^\circ_5$
			*Ti*II	3103·82	50	$b^2G_{4\frac{1}{2}} - x^2F^\circ_{3\frac{1}{2}}$
3102·4	3106·3		*Fe*II	3105·16	5	$c^4D_{1\frac{1}{2}} - x^4D^\circ_{\frac{1}{2}}$
			*Fe*II	3105·55	5	$b^4D_{\frac{1}{2}} - x^4D^\circ_{\frac{1}{2}}$
			*Ti*II	3105·10	20	$b^4P_{\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$
			*Ti*II	3106·24	35	$b^4P_{2\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
			*Sc*II	3107·57	6	$z^1D^\circ_2 - c^1D_2$
3106·7	3110·6		*Ti*II	3110·08	8	$b^2D_{2\frac{1}{2}} - x^2D^\circ_{2\frac{1}{2}}$
			*Ti*II	3110·70	20	$b^4P_{1\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$
			*V*II	3110·71	70	$a^5F_3 - z^5G^\circ_4$
3110·6	3114·6		*Fe*II	3114·29	7	$b^4D_{1\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$
			*Fe*II	3114·68	4	$b^4D_{\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$
3112·7	3116·7		*Fe*II	3116·59	5	$b^4D_{2\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$
			*Ti*II	3117·66	20	$b^4P_{1\frac{1}{2}} - z^4P^\circ_{\frac{1}{2}}$
			*V*II	3118·39	60	$a^5F_2 - z^5G^\circ_3$
3119·4	3123·4		*Cr*II	3125·00	20	$a^4D_{2\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*V*II	3125·29	50	$a^5F_1 - z^5G^\circ_2$
3123·3	3127·3		*V*II	3126·21	25	$a^5F_4 - z^5G^\circ_4$
3124·1	3128·1		*Cr*II	3128·71	5	$a^4D_{1\frac{1}{2}} - z^4F^\circ_{1\frac{1}{2}}$
3127·0	3131·0		*V*II	3130·27	30	$a^5F_3 - z^5P^\circ_3$
			*Ti*II	3130·80	15	$a^4F_{2\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$
			*Fe*II	3131·72	4	$a^2I_{5\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3128·1	3132·1		*Cr*II	3132·06	20	$a^4D_{3\frac{1}{2}} - z^4F^\circ_{4\frac{1}{2}}$
			*V*II	3133·34	25	$a^5F_2 - z^5G^\circ_2$
3129·5	3133·5		*Fe*II	3133·05	4	$b^4D_{1\frac{1}{2}} - x^4D^\circ_{2\frac{1}{2}}$
			*V*II	3134·94	35	$a^3H_6 - y^3H^\circ_6$
3133·2	3137·2	*f*	*V*II	3136·51	30	$a^3H_5 - y^3H^\circ_5$
			*Cr*II	3136·71	5	$a^4D_{2\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$
3144·0	3148·0	*g*	*Cr*II	3147·24	5	$a^4D_{3\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*Ti*II	3148·04	12	$a^4F_{1\frac{1}{2}} - z^2D^\circ_{1\frac{1}{2}}$
3146·5	3150·5	*f*	*Ti*II	3152·26	15	$b^4F_{2\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
3158·5	3162·5	*vg*	*Ti*II	3161·77	30	$b^4F_{2\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*Ti*II	3162·57	35	$b^4F_{3\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Fe*II	3162·80	7	$c^2G_{3\frac{1}{2}} - y^2G^\circ_{3\frac{1}{2}}$
			*Fe*II	3163·09	5	$a^4P_{2\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$
3164·7	3168·7	*g*	*Fe*II	3167·85	11	$b^2G_{3\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$
			*Ti*II	3168·53	40	$b^4F_{4\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
3174·6	3178·6	*br*	*Fe*II	3177·53	10	$b^4D_{3\frac{1}{2}} - y^4D^\circ_{3\frac{1}{2}}$
			*Fe*II	3179·50	9	$c^2D_{2\frac{1}{2}} - y^2F^\circ_{3\frac{1}{2}}$
			*Fe*II	3180·16	7	$c^2D_{1\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$
3182·7	3186·8	*g*	*Fe*II	3186·74	11	$a^4P_{1\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*Fe*II	3187·29	8	$c^2G_{4\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$
			*V*II	3187·71	20	$a^3F_2 - z^3F^\circ_2$
			*V*II	3188·54	25	$a^3F_3 - z^3F^\circ_3$
3186·6	3190·7	*br*	*V*II	3190·68	30	$a^3F_4 - z^3F^\circ_4$
			*Ti*II	3190·85	30	$a^2D_{2\frac{1}{2}} - y^2F^\circ_{3\frac{1}{2}}$
3188·2	3192·3	*g*	*Fe*II	3192·06	4	$b^2G_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$
			*Fe*II	3192·92	9	$a^4P_{2\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Fe*II	3193·81	11	$a^4P_{\frac{1}{2}} - z^4D^\circ_{\frac{1}{2}}$
3192·6	3196·6	*f*	*Fe*II	3196·11	3	$a^4P_{2\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*Cr*II	3197·11	15	$a^4G_{4\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3204·7	3208·8	*br*	*Cr*II	3209·19	10	$a^4G_{3\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$
			*Fe*II	3210·45	10	$a^4P_{\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
3209·5	3213·5	*g*	*Fe*II	3231·31	13	$a^4P_{1\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
3213·5	3217·6	*g*	*Ti*II	3217·07	30	$a^4F_{3\frac{1}{2}} - z^4F^\circ_{4\frac{1}{2}}$
			*V*II	3217·10	30	$b^3G_5 - z^3H^\circ_6$
			*Ti*II	3218·25	25	$a^4P_{2\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$
			*Ti*II	3218·28	25	$a^2H_{4\frac{1}{2}} - y^2G^\circ_{3\frac{1}{2}}$
3219·1	3223·2	*vg*	*Ti*II	3222·85	35	$a^4F_{2\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*Ti*II	3224·25	35	$a^2H_{5\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$
3223·2	3227·4	*s*	*Fe*II	3227·73	13	$a^4P_{2\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
			*Ti*II	3228·62	30	$a^2D_{1\frac{1}{2}} - z^2P^\circ_{\frac{1}{2}}$
3227·6	3231·7	*fg*	*Fe*II	3231·70	5	$b^4D_{2\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$
			*Ti*II	3232·29	30	$a^2G_{3\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$
			*Fe*II	3232·79	6	$c^2G_{3\frac{1}{2}} - z^2F^\circ_{2\frac{1}{2}}$
3230·0	3234·1	*g*	*Ti*II	3234·52	75	$a^4F_{4\frac{1}{2}} - z^4F^\circ_{4\frac{1}{2}}$
3232·3	3236·4	*fg*	*Ti*II	3236·59	70	$a^4F_{3\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*Fe*II	3237·40	5	$b^4D_{1\frac{1}{2}} - y^4F^\circ_{1\frac{1}{2}}$
			*Fe*II	3237·82	9	$b^4D_{\frac{1}{2}} - y^4F^\circ_{1\frac{1}{2}}$
			*V*II	3237·85	25	$b^3G_4 - z^3H^\circ_5$
3234·9	3239·0	*g*	*Ti*II	3239·05	60	$a^4F_{2\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$
			*Ti*II	3239·67	30	$a^2D_{3\frac{1}{2}} - z^2P^\circ_{1\frac{1}{2}}$
3238·9	3243·0	*fg*	*Ti*II	3242·01	60	$a^4F_{1\frac{1}{2}} - z^4F^\circ_{1\frac{1}{2}}$
			*Fe*II	3243·72	8	$c^2G_{4\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$
3244·0	3248·1	*br*	*Fe*II	3247·17	8	$b^4D_{1\frac{1}{2}} - y^4F^\circ_{2\frac{1}{2}}$
			*Fe*II	3247·40	4	$c^2G_{3\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$
			*Ti*II	3248·61	50	$b^4B_{2\frac{1}{2}} - y^4D^\circ_{3\frac{1}{2}}$
3247·9	3252·0	*s*	*Ti*II	3251·85 ·94	30	$a^4F_{2\frac{1}{2}} - z^4F^\circ_{1\frac{1}{2}}$
			*Ti*II	3252·85	40	$a^2D_{2\frac{1}{2}} - y^2D^\circ_{2\frac{1}{2}}$
			*Ti*II	3252·85	40	$a^4F_{3\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3250·3	3254·4	*g*	*Ti*II	3254·26	30	$a^4F_{4\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
			*V*II	3254·76	25	$b^3G_3 - z^3H^\circ_4$
			*Fe*II	3255·90	9	$a^4D_{3\frac{1}{2}} - z^6D^\circ_{3\frac{1}{2}}$
3254·4	3258·6	*fg*	*Fe*II	3258·77	10	$b^4D_{2\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$
			*Fe*II	3259·05	10	$b^4D_{3\frac{1}{2}} - y^4F^\circ_{4\frac{1}{2}}$
3257·3	3261·4	*fg*	*Ti*II	3261·58	60	$b^4P_{1\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$
			*Ti*II	3261·58	60	$b^2G_{4\frac{1}{2}} - z^2H^\circ_{5\frac{1}{2}}$
3259·9	3264·1	*f*	*Ti*II	3263·69	4	$a^4P_{1\frac{1}{2}} - z^4S^\circ_{1\frac{1}{2}}$
3263·3	3267·4	*w*	*V*II	3267·71	50	$a^3F_2 - z^3G^\circ_3$
3267·5	3271·7	*vg*	*V*II	3271·14	60	$a^3F_3 - z^3G^\circ_4$
			*Ti*II	3271·67	25	$b^4P_{2\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$
			*Ti*II	3272·09	25	$b^4P_{\frac{1}{2}} - y^4D^\circ_{1\frac{1}{2}}$
3272·7	3276·9	*s*	*V*II	3276·14	70	$a^3F_4 - z^3G^\circ_5$
			*Fe*II	3276·60	5	$b^2F_{2\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$
			*Fe*II	3277·34	10	$a^4D_{3\frac{1}{2}} - z^6D^\circ_{4\frac{1}{2}}$
			*Ti*II	3278·30	30	$b^4P_{1\frac{1}{2}} - y^4D^\circ_{1\frac{1}{2}}$
3277·5	3281·7	*f*	*Fe*II	3281·29	8	$a^4D_{2\frac{1}{2}} - z^6D^\circ_{2\frac{1}{2}}$
			*Ti*II	3282·33	25	$b^4P_{\frac{1}{2}} - y^4D^\circ_{\frac{1}{2}}$
3283·8	3287·9	*g*	*Ti*II	3287·67	40	$b^2G_{3\frac{1}{2}} - z^2H^\circ_{4\frac{1}{2}}$
			*Ti*II	3288·43	5	$b^4P_{2\frac{1}{2}} - y^4D^\circ_{1\frac{1}{2}}$
			*Ti*II	3288·58	5	$b^4P_{1\frac{1}{2}} - y^4D^\circ_{\frac{1}{2}}$
			*Fe*II	3289·34	7	$b^2G_{3\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$
3287·4	3291·6	*w*	*Cr*II	3291·77	3	$a^2P_{\frac{1}{2}} - z^2S^\circ_{\frac{1}{2}}$
3291·5	3295·7	*f*	*Fe*II	3295·23	4	$b^4D_{2\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$
			*Cr*II	3295·44	5	$a^4G_{5\frac{1}{2}} - z^4H^\circ_{6\frac{1}{2}}$
			*Fe*II	3295·81	6	$a^4D_{1\frac{1}{2}} - z^6D^\circ_{1\frac{1}{2}}$
3298·8	3303·0	*vg*	*Fe*II	3302·86	4	$a^4D_{2\frac{1}{2}} - z^6D^\circ_{3\frac{1}{2}}$
			*Fe*II	3303·46	4	$a^4D_{\frac{1}{2}} - z^6D^\circ_{\frac{1}{2}}$
3302·9	3307·1	*w*	*Cr*II	3307·04	8	$a^4G_{4\frac{1}{2}} - z^4H^\circ_{5\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3307·4	3311·6	*w*	*Cr*II	3311·94	3	$a^4G_{3\frac{1}{2}} - z^4H^\circ_{4\frac{1}{2}}$
3313·9	3318·1	*s*	*Ti*II	3318·03	10	$b^4F_{2\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
3318·3	3322·5	*s*	*Ti*II	3321·71	25	$b^4P_{1\frac{1}{2}} - z^4S^\circ_{1\frac{1}{2}}$
			*Ti*II	3322·93	75	$b^4F_{4\frac{1}{2}} - z^4F^\circ_{4\frac{1}{2}}$
			*Fe*II	3323·07	8	$b^2F_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$
3324·8	3329·0	*fg*	*Ti*II	3329·44, ·52	70	$b^4F_{3\frac{1}{2}} - z^4F^\circ_{3\frac{1}{2}}$
3327·5	3331·8	*w*	*Ti*II	3332·11	30	$b^4P_{2\frac{1}{2}} - z^4S^\circ_{1\frac{1}{2}}$
3331·3	3335·5	*vg*	*Ti*II	3335·17, ·22	40	$b^4F_{2\frac{1}{2}} - z^4F^\circ_{2\frac{1}{2}}$
			*Cr*II	3336·35	5	$a^4D_{\frac{1}{2}} - y^4P^\circ_{\frac{1}{2}}$
3336·6	3340·8	*s*	*Cr*II	3339·80	10	$a^4D_{1\frac{1}{2}} - y^4P^\circ_{1\frac{1}{2}}$
			*Ti*II	3340·33, ·39	35	$b^4F_{1\frac{1}{2}} - z^4F^\circ_{1\frac{1}{2}}$
			*Ti*II	3341·84	50	$a^2F_{2\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$
3344·6	3348·8	*vs*	*Ti*II	3349·00	75	$a^2F_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$
			*Ti*II	3349·39, ·46	125	$a^4F_{4\frac{1}{2}} - z^4G^\circ_{5\frac{1}{2}}$
3356·4	3360·7	*s*	*Cr*II	3360·31	20	$b^4D_{3\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
			*Ti*II	3361·19	125	$a^4F_{3\frac{1}{2}} - z^4G^\circ_{4\frac{1}{2}}$
3359·6	3363·9	*vw*	*Cr*II	3363·72	2	$a^4D_{1\frac{1}{2}} - z_4P^\circ_{2\frac{1}{2}}$
3363·3	3367·6	*s*	*Ti*II	3366·18	8	$a^2P_{1\frac{1}{2}} - z^2P^\circ_{\frac{1}{2}}$
			*Cr*II	3368·06	20	$a^4D_{3\frac{1}{2}} - y^4P^\circ_{2\frac{1}{2}}$
			*Sc*II	3368·95	10	$a^3D_2 - z^3P^\circ_1$
3368·5	3372·8	*vg*	*Sc*II	3372·18	10	$a^3D_3 - z^3P^\circ_2$
			*Ti*II	3372·77, ·86	100	$a^4F_{2\frac{1}{2}} - z^4G^\circ_{3\frac{1}{2}}$
3375·1	3379·4	*vg*	*Cr*II	3378·34	5	$b^4D_{3\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Cr*II	3379·38	3	$b^4D_{1\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$
			*Cr*II	3379·82	5	$b^4D_{2\frac{1}{2}} - z^4D^\circ_{2\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
			*Ti*II	3380·26 3380·30	30	$a^4F_{4\frac{1}{2}} - z^4G^\circ_{4\frac{1}{2}}$
			*Sr*II	3380·75	50	$5^2P^\circ_{\frac{1}{2}} - 5^2D_{1\frac{1}{2}}$
3378·7	3383·0	*vg*	*Cr*II	3382·69	10	$a^4D_{2\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
			*Ti*II	3383·77	125	$a^4F_{1\frac{1}{2}} - z^4G^\circ_{2\frac{1}{2}}$
3383·7	3388·0	*g*	*Ti*II	3387·85	50	$a^4F_{3\frac{1}{2}} - z^4G^\circ_{3\frac{1}{2}}$
			*Ti*II	3388·76	8	$a^2P_{1\frac{1}{2}} - y^2D^\circ_{2\frac{1}{2}}$
3389·7	3394·0	*g*	*Cr*II	3393·02	5	$b^4D_{\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*Cr*II	3393·85	4	$b^4D_{1\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*Cr*II	3394·30	4	$b^4D_{2\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
			*Ti*II	3394·55	40	$a^4F_{2\frac{1}{2}} - z^4G^\circ_{2\frac{1}{2}}$
3394·2	3398·5	*fg*	*Fe*II	3398·35	4	$a^2I_{6\frac{1}{2}} - z^2I^\circ_{6\frac{1}{2}}$
3399·0	3403·3	*vg*	*Ti*II	3402·42	8	$a^2P_{\frac{1}{2}} - y^2D^\circ_{1\frac{1}{2}}$
			*Cr*II	3402·42	4	$b^4D_{\frac{1}{2}} - z^4D^\circ_{\frac{1}{2}}$
			*Cr*II	3403·35	15	$b^4D_{1\frac{1}{2}} - z^4D^\circ_{\frac{1}{2}}$
3404·4	3408·7	*f*	*Cr*II	3408·78	20	$a^4D_{3\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3413·2	3417·6	*g*	*Fe*II	3416·02	5	$a^2P_{1\frac{1}{2}} - z^4P^\circ_{\frac{1}{2}}$
3416·9	3421·2	*fg*	*Cr*II	3421·22	10	$a^4D_{\frac{1}{2}} - z^4P^\circ_{\frac{1}{2}}$
			*Cr*II	3422·76	20	$a^4D_{2\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$
3423·5	3427·9	*f*				
3428·9	3433·3	*f*	*Cr*II	3433·32	5	$a^4D_{1\frac{1}{2}} - z^4P^\circ_{\frac{1}{2}}$
3432·8	3437·1	*g*	*Fe*II	3436·11	5	$b^2F_{3\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$
3434·5	3438·9	*f*	*Mn*II	3438·96	3	$a^5S_2 - z^7P^\circ_3$
3440·0	3444·4	*w*	*Ti*II	3444·33	30	$b^4F_{4\frac{1}{2}} - z^4G^\circ_{5\frac{1}{2}}$
3451·2	3455·6	*vg*	*Ti*II	3456·39	20	$b^2P_{1\frac{1}{2}} - y^2P^\circ_{1\frac{1}{2}}$
			*Fe*II	3456·93	5	$b^4D_{3\frac{1}{2}} - y^4P^\circ_{2\frac{1}{2}}$
3456·1	3460·5	*g*	*Mn*II	3460·33	20	$a^5D_3 - z^5P^\circ_2$
			*Ti*II	3461·50	20	$b^4F_{3\frac{1}{2}} - z^4G^\circ_{4\frac{1}{2}}$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3460·7	3465·1	*w*	*Ti*II	3465·65	3	$b^2P_{1\frac{1}{2}} - y^2P^\circ_{\frac{1}{2}}$
3464·5	3468·9	*f*	*Fe*II	3468·68	8	$c^2G_{3\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$
3469·5	3473·9	*f*	*Mn*II	3473·97, 74·96	8	$a^5D_3 - z^5P^\circ_3$
				3474·15	8	$a^5D_2 - z^5P^\circ_1$
3473·5	3478·0	*w*	*Ti*II	3477·19	15	$b^4F_{2\frac{1}{2}} - z^4G^\circ_{3\frac{1}{2}}$
3478·9	3483·4	*f*	*Mn*II	3482·91	12	$a^5D_2 - z^5P^\circ_2$
			*Cr*II	3484·16	3	$a^4D_{2\frac{1}{2}} - z^6P^\circ_{2\frac{1}{2}}$
3484·6	3489·0	*g*	*Fe*II	3487·99	4	$a^4P_{1\frac{1}{2}} - z^6F^\circ_{2\frac{1}{2}}$
			*Mn*II	3488·68	10	$a^5D_1 - z^5P^\circ_1$
3487·1	3491·5	*w*	*Ti*II	3491·06	10	$b^4F_{1\frac{1}{2}} - z^4G^\circ_{2\frac{1}{2}}$
3491·3	3495·7	*w*	*Fe*II	3493·47	9	$c^2G_{4\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$
			*Fe*II	3494·67	5	$a^2P_{1\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3501·0	3505·4	*fg*	*Ti*II	3504·88, ·92	80	$b^2G_{4\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$
3506·3	3510·8	*fg*	*Ti*II	3510·85	60	$b^2G_{3\frac{1}{2}} - y^2G^\circ_{3\frac{1}{2}}$
			*Cr*II	3511·84	4	$a^4D_{3\frac{1}{2}} - z^6P^\circ_{2\frac{1}{2}}$
3526·4	3530·8	*w*	*V*II	3530·78	25	$a^3F_2 - y^3D^\circ_1$
3530·8	3535·3	*fg*	*Ti*II	3535·41	40	$b^2P_{\frac{1}{2}} - x^2D^\circ_{2\frac{1}{2}}$
			*Sc*II	3535·73	15	$a^1D_2 - z^1F^\circ_3$
3552·5	3557·0	*w*	*V*II	3556·80	40	$a^3F_4 - y^3D^\circ_3$
			*Sc*II	3558·53	20	$a^3D_2 - z^3D^\circ_3$
3568·7	3573·2	*w*	*Sc*II	3572·48, ·57	50	$a^3D_3 - z^3D^\circ_3$
			*Ti*II	3573·73	20	$a^2F_{2\frac{1}{2}} - z^4D^\circ_{1\frac{1}{2}}$
3572·2	3576·7	*w*	*Sc*II	3576·33, ·39	30	$a^3D_2 - z^3D^\circ_2$
3576·8	3581·4	*fg*	*Sc*II	3580·93	20	$a^3D_1 - z^3D^\circ_1$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3581·5	3586·1	*f*	*Cr*II	3585·29	4	$a^4P_{2\frac{1}{2}} - y^4P^\circ_{2\frac{1}{2}}$
			*Cr*II	3585·52	3	$a^4P_{1\frac{1}{2}} - y^4P^\circ_{2\frac{1}{2}}$
			*Ti*II	3587·14	12	$a^2F_{3\frac{1}{2}} - z^4D^\circ_{3\frac{1}{2}}$
3609·7	3614·3	*fg*	*Sc*II	3613·81, ·88	100	$a^3D_3 - z^3F^\circ_4$
			*Fe*II	3614·87	4	$c^2G_{3\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$
3615·7	3620·3	*w*	*Fe*II	3621·27	4	$a^2S_{\frac{1}{2}} - z^2P^\circ_{1\frac{1}{2}}$
3621·2	3625·9	*fg*	*Ti*II	3624·84	70	$a^2P_{\frac{1}{2}} - z^2S^\circ_{\frac{1}{2}}$
3621·1	3631·7	*fg*	*Sc*II	3630·74, ·78	100	$a^3D_2 - z^3F^\circ_3$
			*Cr*II	3631·71	10	$a^4P_{1\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$
3637·9	3642·5	*fg*	*Ti*II	3641·33	100	$a^2P_{1\frac{1}{2}} - z^2S^\circ_{\frac{1}{2}}$
			*Sc*II	3642·77, ·83	50	$a^3D_1 - z^3F^\circ_2$
3657·4	3662·0	*fg*	*Ti*II	3662·24	40	$b^2D_{1\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$
3669·6	3674·3	*w*				
3675·1	3679·7	*vg*				
3680·5	3685·2	*g*	*Ti*II	3685·20	250	$a^2F - z^2D^\circ$
3687·4	3697·0	*vw*	*H*17	3697·1		$2^2P^\circ - 17^2D$
3694·5	3704·1	*vw*	*H*16	3703·8		$2^2P^\circ - 16^2D$
3702·9	3707·6		*Ti*II	3706·22	20	$b^2D_{1\frac{1}{2}} - z^2P^\circ_{1\frac{1}{2}}$
	3712·6	*vw*	*H*15	3711·97		$2^2P^\circ - 15^2D$
3711·4	3721·1	*w*	*H*14	3721·1		$2^2P^\circ - 14^2D$
3717·3	3722·0	*w*	*Ti*II	3721·64	15	$a^2F_{2\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$
3725·4	3735·1	*w*	*H*13	3734·37		$2^2P^\circ - 13^2D$
3730·5	3735·3	*w*				
3738·8	3743·6	*br*	*Ti*II	3741·65	50	$b^2D_{2\frac{1}{2}} - y^2D^\circ_{2\frac{1}{2}}$
	3748·6		*H*12	3750·15		$2^2P^\circ - 12^2D$

λ meas.	λ corr.	Qu.	Origin	λ Lab.	I Lab.	Multiplet
3753·1	3757·8	*s*	*Ti*II	3757·69	30	$b^2D_{1\frac{1}{2}} - y^2D^\circ_{1\frac{1}{2}}$
			*Ti*II	3759·30	200	$a^2F_{3\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$
			*Ti*II	3761·32	200	$a^2F_{2\frac{1}{2}} - z^2F^\circ_{2\frac{1}{2}}$
3771·7	3776·4	*vw*	*Ti*II	3776·06	6	$b^2D_{2\frac{1}{2}} - y^2D^\circ_{1\frac{1}{2}}$
3777·5	3782·3	*vw*				
3787·7	3797·7	*s*	*H*10	3797·90		$2^2P^\circ - 10^2D$

SOME NEW IDENTIFICATIONS OF Fe^+ LINES IN THE SOLAR SPECTRUM.

J. C. Dobbie, Ph.D.

As the result of an analysis of the polar line spectrum of iron undertaken at the Solar Physics Observatory, Cambridge, many new terms in the spectrum of Fe^+ have been identified. Although most of the well-known Fe^+ lines in stellar spectra have previously been classified by means of the terms published by Russell * some of the new terms, by their combinations with one another or with Russell's b^4D term, account for a number of the fainter lines of the solar spectrum.

The identifications are given in Table I. Column (1) gives the Cambridge measured wave-lengths at the pole of the arc; (2) the wave-lengths in the Sun, as given by the Revised Rowland; (3) the intensities in the arc estimated on an arbitrary scale; (4) the solar intensities; (5) the term combinations; and (6) the present identifications given in the Revised Rowland.

The extent to which the lines in this table account for the solar lines cannot be definitely decided until an examination is made of the multiplets of the other elements present. This work is not undertaken here as it is understood that Miss Moore is making a detailed examination of all the observed and predicted Fe^+ lines present in the solar spectrum.

It should be noted that the low level lines, *i.e.* those for which the upper state is a z^4P^0, z^4D^0 or z^4F^0 level, are stronger in the Sun relative to the pole of the arc than are the other lines, all of which come from levels considerably higher.

The stronger Fe^+ lines, which have not yet been classified but which should evidently be present in the Sun, are given with the nearest solar line in Table II.

* "Multiplets in the Spark Spectrum of Iron," *Ap. J.*, **64**, 194, 1926.

TABLE I

(1) Pole of Arc	(2) Sun	(3) Arc Intensity	(4) Solar Intensity	(5) Multiplet	(6) Solar Identification
2980·963	966	4	−1*N*	$a^2F_{2\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$	
82·059	076	7	−1	$b^2D_{1\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$	*Fe?*
82·239	236	4	−2*N*	$b^2G_{3\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$	*Co?*
86·618	646	5	−1*N*	$a^2F_{2\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$	*Fe*
97·299	336	7	2	$b^2D_{2\frac{1}{2}} - y^2F^\circ_{3\frac{1}{2}}$	*Ca* − *Cu?*
3000·060	078	5	−1	$b^2G_{3\frac{1}{2}} - y^4G^\circ_{2\frac{1}{2}}$	*Fe⁺?*
02·330	333	5	−2	$b^2F_{2\frac{1}{2}} - y^2G^\circ_{3\frac{1}{2}}$	
36·987	983	5	−1	$z^4P^\circ_{2\frac{1}{2}} - e^4D_{2\frac{1}{2}}$	
49·013	021	5	0	$z^4P^\circ_{1\frac{1}{2}} - e^4D_{1\frac{1}{2}}$	
56·802	796	5	1	$a^2I_{6\frac{1}{2}} - x^4G^\circ_{5\frac{1}{2}}$	*Fe⁺?*
62·234	252	9	4*N*	$a^2I_{5\frac{1}{2}} - z^2H^\circ_{4\frac{1}{2}}$	*Co* −
65·315	317	5	1	$b^2F_{2\frac{1}{2}} - z^2F^\circ_{2\frac{1}{2}}$	*Pd* −
70·692	696	4	0	$b^2G_{4\frac{1}{2}} - x^4D^\circ_{3\frac{1}{2}}$	
71·141	146	4	1	$z^4P^\circ_{\frac{1}{2}} - e^4P_{1\frac{1}{2}}$	*OH*
75·235	240	2	3	$b^2G_{3\frac{1}{2}} - x^4D^\circ_{2\frac{1}{2}}$	*Ti⁺*
76·455	436	6	1	$z^4P^\circ_{1\frac{1}{2}} - e^4D_{2\frac{1}{2}}$	
77·168	189	10	3	$a^2I_{6\frac{1}{2}} - z^2H^\circ_{5\frac{1}{2}}$	*Fe⁺*
78·698	663	8	8*d?*	$z^4P^\circ_{2\frac{1}{2}} - e^4D_{3\frac{1}{2}}$	*Ti⁺*
80·405	423	2	−1	$a^2I_{5\frac{1}{2}} - z^2H^\circ_{5\frac{1}{2}}$	
83·024	043	2	1	$b^2F_{3\frac{1}{2}} - z^2F^\circ_{2\frac{1}{2}}$	
89·388	404	4	1	$c^2D_{2\frac{1}{2}} - x^2G^\circ_{3\frac{1}{2}}$	*Ti⁺*
96·295	325	5	0	$b^2F_{3\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$	− *OH*
97·415	434	2	−1	$b^2F_{2\frac{1}{2}} - y^4G^\circ_{2\frac{1}{2}}$	
3105·164	175	5	1	$b^4D_{1\frac{1}{2}} - x^4D^\circ_{\frac{1}{2}}$	*Fe⁺*
05·547	564	5	1	$b^4D_{\frac{1}{2}} - x^4D^\circ_{\frac{1}{2}}$	
06·558	560	4	2	$b^2G_{3\frac{1}{2}} - x^4D^\circ_{3\frac{1}{2}}$	*Zr⁺*
14·294	317	7	2	$b^4D_{1\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$	*Fe⁺?*
14·679	674	4	0	$b^4D_{\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$	
16·587	504	5	1	$b^4D_{2\frac{1}{2}} - x^4D^\circ_{1\frac{1}{2}}$	*Fe⁺?*
20·024	013	1	−1*N*	$b^2F_{3\frac{1}{2}} - y^4G^\circ_{3\frac{1}{2}}$	
29·013	008	1	−2	$b^2F_{3\frac{1}{2}} - y^4G^\circ_{4\frac{1}{2}}$	*Co*
30·561	568	2	1	$b^2G_{4\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	*OH*
3131·718	709	4	0	$a^2I_{5\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$	
33·047	067	4	1	$b^4D_{1\frac{1}{2}} - x^4D^\circ_{2\frac{1}{2}}$	*Sc⁺?*
35·359	357	9	1	$b^4D_{2\frac{1}{2}} - x^4D^\circ_{2\frac{1}{2}}$	*Fe⁺*
44·750	738	5	1	$b^4D_{3\frac{1}{2}} - x^4D^\circ_{2\frac{1}{2}}$	*Ti⁺V⁺*

(1) Pole of Arc	(2) Sun	(3) Arc Intensity	(4) Solar Intensity	(5) Multiplet	(6) Solar Identification
46·748	757	2	0	$b^2G_{4\frac{1}{2}} - y^4F^\circ_{4\frac{1}{2}}$	
54·201	201	11	3	$b^2G_{4\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$	Fe^+Ti^+
55·951	941	2	0	$b^2G_{4\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	Zr^+ –
62·799	804	7	0	$c^2G_{3\frac{1}{2}} - y^2G^\circ_{3\frac{1}{2}}$	Fe^+
67·853	911	11	2	$b^2G_{3\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	Fe^+
75·077	046	4	0	$c^2D_{2\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$	Fe
77·531	543	10	2	$b^4D_{3\frac{1}{2}} - x^4D^\circ_{3\frac{1}{2}}$	$-Fe^+$
79·505	514	9	1*N*	$c^2D_{2\frac{1}{2}} - y^2F^\circ_{3\frac{1}{2}}$	Fe
80·163	122 237	7	−1*N* 3	$c^2D_{1\frac{1}{2}} - y^2F^\circ_{2\frac{1}{2}}$	$\underline{Fe} - Co$
85·100	093	1	−3	$b^2G_{3\frac{1}{2}} - y^4F^\circ_{2\frac{1}{2}}$	*Mo?*
87·295	309	8	1	$c^2G_{4\frac{1}{2}} - y^2G^\circ_{4\frac{1}{2}}$	Fe^+
91·375	315 415	1	−2 0	$b^4D_{1\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$	*Co*
92·060	040	4	−1	$b^2G_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$	
3203·512	513	1	−2	$b^4D_{3\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$	$Cr^+?$
09·605	625	1	−2	$b^2D_{1\frac{1}{2}} - y^2D^\circ_{1\frac{1}{2}}$	
26·381	348 447	2	−3 −1	$c^2F_{2\frac{1}{2}} - w^2F^\circ_{2\frac{1}{2}}$	*OH*
30·496	473	1	0*N*	$b^2F_{3\frac{1}{2}} - x^4D^\circ_{3\frac{1}{2}}$	
31·704	708	5	1	$b^4D_{2\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	Zr^+
32·793	798	6	−1	$c^2G_{3\frac{1}{2}} - z^2F^\circ_{2\frac{1}{2}}$	$Ti - \underline{Fe}$
37·404	432	5	1*N*	$b^4D_{1\frac{1}{2}} - y^4F^\circ_{1\frac{1}{2}}$	Fe
37·817	851	9	2*Nd?*	$b^4D_{\frac{1}{2}} - y^4F^\circ_{1\frac{1}{2}}$	V^+
41·689	689	2	−1	$b^4D_{3\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	
43·725	766	8	2	$c^2G_{4\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$	$Fe^+ - Mn$
47·173	193	8	3	$b^4D_{1\frac{1}{2}} - y^4F^\circ_{2\frac{1}{2}}$	$Co - Fe^+$
47·397	407	4	0	$c^2G_{3\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$	
49·659	636 685	4	−1 0	$b^4D_{2\frac{1}{2}} - y^4F^\circ_{2\frac{1}{2}}$	
49·914	928	1	−2	$b^4D_{1\frac{1}{2}} - z^2D^\circ_{1\frac{1}{2}}$	
57·360	361	1	−1	$b^2F_{3\frac{1}{2}} - y^4D^\circ_{2\frac{1}{2}}$	
57·894	908	4	0	$c^2F_{3\frac{1}{2}} - w^2F^\circ_{3\frac{1}{2}}$	V^+
3258·773	784	10	3	$b^4D_{2\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	$Pd\underline{Fe}^+$
59·048	063	10	3	$b^4D_{3\frac{1}{2}} - y^4F^\circ_{4\frac{1}{2}}$	Fe^+
66·937	951	5	1	$b^2G_{4\frac{1}{2}} - z^4H^\circ_{4\frac{1}{2}}$	$Fe^+?$
67·034	063	4	1	$b^2D_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$	OH
68·510	522	4	−1	$c^2G_{3\frac{1}{2}} - y^4G^\circ_{2\frac{1}{2}}$	
69·771	768	4	1	$c^2G_{4\frac{1}{2}} - y^4G^\circ_{3\frac{1}{2}}$	Ti^+
73·497	483	4	1*N*	$c^2G_{3\frac{1}{2}} - y^4G^\circ_{3\frac{1}{2}}$	

(1) Pole of Arc	(2) Sun	(3) Arc Intensity	(4) Solar Intensity	(5) Multiplet	(6) Solar Identification
76·604	618	5	1	$b^2F_{2\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	
79·646	660	2	0	$c^2G_{4\frac{1}{2}} - y^4G^\circ_{4\frac{1}{2}}$	
89·342	373	7	4	$b^2G_{3\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$	V^+
95·233	249	4	1	$b^4D_{2\frac{1}{2}} - y^4D^\circ_{3\frac{1}{2}}$	$-Ce^+?$
97·880	883	5	1	$b^2F_{2\frac{1}{2}} - z^2D^\circ_{1\frac{1}{2}}$	Fe
3304·432	451	1	−2	$b^2F_{2\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	
23·066	085	8	1	$b^2F_{3\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$	Fe^+
25·012	013	1	0	$b^2F_{3\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	
38·522	521	3	−1	$b^2D_{2\frac{1}{2}} - y^4P^\circ_{1\frac{1}{2}}$	Co
58·252	283	3	3	$b^4D_{2\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$	Ti
60·102	125	3	0	$a^2I_{5\frac{1}{2}} - z^2I^\circ_{5\frac{1}{2}}$	$NH -$
65·414	447	1	0	$b^4D_{2\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$	$-NH$
66·959	983	3	−2	$c^2F_{3\frac{1}{2}} - x^2D^\circ_{2\frac{1}{2}}$	
68·626	659	0	−1	$c^2F_{2\frac{1}{2}} - x^2D^\circ_{2\frac{1}{2}}$	NH
81·004	066	4	−3	$c^2F_{2\frac{1}{2}} - x^2D^\circ_{1\frac{1}{2}}$	NH
87·326	309	2	0	$b^2F_{3\frac{1}{2}} - z^4I^\circ_{4\frac{1}{2}}$	
88·135	176	2	3	$b^4D_{3\frac{1}{2}} - z^4H^\circ_{4\frac{1}{2}}$	Co
95·336	371	4	3	$c^2G_{3\frac{1}{2}} - x^4D^\circ_{3\frac{1}{2}}$	
98·355	378	4	0	$a^2I_{6\frac{1}{2}} - z^2I^\circ_{6\frac{1}{2}}$	
3414·144	138	1	−1N	$b^2F_{2\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$	
16·021	033	5	3	$a^2P_{1\frac{1}{2}} - z^4P^\circ_{\frac{1}{2}}$	
36·113	115	5	0	$b^2F_{3\frac{1}{2}} - z^2D^\circ_{2\frac{1}{2}}$	
42·239	233	5	−1	$b^2F_{3\frac{1}{2}} - z^4G^\circ_{2\frac{1}{2}}$	
48·434	463	1	−2N	$b^2F_{3\frac{1}{2}} - z^4H^\circ_{4\frac{1}{2}}$	
56·929	935	5	1	$b^4D_{3\frac{1}{2}} - y^4P^\circ_{2\frac{1}{2}}$	Co
64·497	475	3	1	$c^2G_{4\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	Sr^+
68·680	687	8	1	$c^2G_{3\frac{1}{2}} - z^2G^\circ_{3\frac{1}{2}}$	Fe^+
70·242	245	2	−2N	$b^2F_{3\frac{1}{2}} - z^4G^\circ_{4\frac{1}{2}}$	
84·347	344	1	−2d?	$c^2G_{4\frac{1}{2}} - y^4F^\circ_{4\frac{1}{2}}$	
93·469	480	9	2	$c^2G_{4\frac{1}{2}} - z^2G^\circ_{4\frac{1}{2}}$	
94·672	677	5	2	$a^2P_{1\frac{1}{2}} - z^4P^\circ_{2\frac{1}{2}}$	Fe
95·616	664	2	3	$c^2G_{4\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	Co
99·877	875	4	0	$c^2G_{3\frac{1}{2}} - y^4F^\circ_{3\frac{1}{2}}$	
3507·386	405	3	1	$a^2P_{\frac{1}{2}} - z^4P^\circ_{1\frac{1}{2}}$	Fe
35·627	625	1	−2	$b^4D_{2\frac{1}{2}} - z^4S^\circ_{1\frac{1}{2}}$	$-CN$
66·155	176	1	2N	$b^2D_{2\frac{1}{2}} - z^2F^\circ_{3\frac{1}{2}}$	Cr, V^+
3614·872	892	4	−1	$c^2G_{3\frac{1}{2}} - z^4H^\circ_{3\frac{1}{2}}$	$Zr^+?$

(1) Pole of Arc	(2) Sun	(3) Arc Intensity	(4) Solar Intensity	(5) Multiplet	(6) Solar Identification
21·273	262	4	0	$a^2S_{\frac{1}{2}} - z^2P^{\circ}_{1\frac{1}{2}}$	
24·890	840	3	5	$a^2S_{\frac{1}{2}} - z^2P^{\circ}_{\frac{1}{2}}$	*Fe – Ti⁺*
32·289	300	1	– 1	$c^2G_{4\frac{1}{2}} - z^4H^{\circ}_{4\frac{1}{2}}$	
3725·305	308	1	0	$b^2D_{1\frac{1}{2}} - y^4D^{\circ}_{1\frac{1}{2}}$	
55·564	575	2	– 1	$c^2D_{1\frac{1}{2}} - z^2P^{\circ}_{1\frac{1}{2}}$	*CN?*
59·461	475	4	0	$c^2D_{1\frac{1}{2}} - z^2P^{\circ}_{\frac{1}{2}}$	
62·895	872	3	– 1*N*	$d^2D_{2\frac{1}{2}} - x^2D^{\circ}_{2\frac{1}{2}}$	*CN*
81·510	518	1	– 1	$b^2D_{2\frac{1}{2}} - y^4D^{\circ}_{2\frac{1}{2}}$	
83·348	351*m*	3	2	$a^2P_{1\frac{1}{2}} - z^4D^{\circ}_{2\frac{1}{2}}$	
3814·122	124*m*		0	$c^2D_{1\frac{1}{2}} - z^2F^{\circ}_{2\frac{1}{2}}$	
27·081	086*w*	0	0	$c^2D_{2\frac{1}{2}} - z^2F^{\circ}_{3\frac{1}{2}}$	
3906·037	034	5	– 1	$c^2F_{2\frac{1}{2}} - x^2G^{\circ}_{3\frac{1}{2}}$	
35·942	981*w*	6	2	$c^2F_{3\frac{1}{2}} - x^2G^{\circ}_{4\frac{1}{2}}$	*Co*
38·970	972	4	– 1	$d^2D_{1\frac{1}{2}} - x^2F^{\circ}_{2\frac{1}{2}}$	
75·029	059	1	– 1*d?*	$d^2D_{2\frac{1}{2}} - y^2P^{\circ}_{1\frac{1}{2}}$	*Ce⁺*
4002·548	509	3	0*d*	$d^2D_{2\frac{1}{2}} - x^2F^{\circ}_{3\frac{1}{2}}$	*Ti*
24·549	578*w*	5	3	$b^2D_{2\frac{1}{2}} - z^2D^{\circ}_{2\frac{1}{2}}$	*Ti*
4446·252	248	1	– 1	$d^2D_{2\frac{1}{2}} - x^2G^{\circ}_{3\frac{1}{2}}$	
4549·212	193	5	– 1	$d^2D_{1\frac{1}{2}} - y^2F^{\circ}_{2\frac{1}{2}}$	
4625·912	923	2	0*N*	$d^2D_{2\frac{1}{2}} - y^2F^{\circ}_{2\frac{1}{2}}$	*Cr*
35·327	316	6	0	$d^2D_{2\frac{1}{2}} - y^2F^{\circ}_{3\frac{1}{2}}$	*Fe⁺*
4953·981	4·005	0	– 3	$c^2F_{2\frac{1}{2}} - y^2G^{\circ}_{3\frac{1}{2}}$	
5019·479	489	0	– 3	$c^2F_{3\frac{1}{2}} - y^2G^{\circ}_{4\frac{1}{2}}$	
5127·866	884	1	– 3	$c^2F_{2\frac{1}{2}} - z^2F^{\circ}_{2\frac{1}{2}}$	(*C* –)
60·824	838	1	– 3*N*	$c^2F_{3\frac{1}{2}} - z^2F^{\circ}_{3\frac{1}{2}}$	
5272·414	406	3	– 2	$d^2D_{2\frac{1}{2}} - y^2D^{\circ}_{2\frac{1}{2}}$	
78·958	962	0	– 2	$d^2D_{1\frac{1}{2}} - x^4F^{\circ}_{1\frac{1}{2}}$	
5408·848	823	1	– 2	$d^2D_{2\frac{1}{2}} - x^4F^{\circ}_{2\frac{1}{2}}$	
5534·862	849*m*	4	2	$b^2H_{5\frac{1}{2}} - z^4F^{\circ}_{4\frac{1}{2}}$	*Fe⁺*

TABLE II

Pole of Arc	Sun	Arc Intensity	Solar Intensity	Solar Identification
3118·645	657	2	2	*Cr*+
3356·265	238	2	−2*d*?	
90·083	109	2	−2*d*?	
3451·228	235	2	−3	
51·318	343	2	0	
82·426	452	2	−2	
3515·817	808	2	−3*N*	
3960·896	917	3	−1	*Ce*+
4024·222	226	5	−2	
66·330	375*w*	2	2	*Co*
4314·289	320	4	1	*Ti*
57·573	519	4	0*Nd*?	*Cr*
61·248	247	2	−1	
4451·546	590*m*	5	3	*Mn*
55·259	322*m*	3	2	*Mn* − *Ti*
80·690	706	2	−3	
4579·522	514	2	−1*d*?	*Co*?
4704·623	677	3	−3	

AN INDEX TO SOUTHERN METEOR SHOWERS.

R. A. McIntosh.

Meteor observers in the southern hemisphere have been handicapped up to the present by not knowing at the outset of a night's work what meteor radiants may be expected to be in activity, and consequently to what portions of the sky attention should be directed for best results. Thus, except on a few occasions when the more prominent annual showers are active, a very large area of sky has to be watched and considerable time elapses before the centres of radiation in activity become apparent.

The need for some guide as to the more prominent meteor showers in the southern hemisphere has been felt ever since the first pioneering observations were made. After eight years of activity the Meteor Section of the New Zealand Astronomical Society has accumulated sufficient data to enable a preliminary outline of southern meteoric activity to be published.

While the present index is not to be regarded as in any sense final and complete, it is hoped that it will prove of immediate benefit to observers in enabling them to plan their work in advance in an effort to determine the durations of the many showers listed here for the first time, their exact positions in the sky and the dates of their maxima.

The 320 radiation centres given in the table represent the mean positions of what are believed to be the principal meteor showers situated on or south of the celestial equator. These positions have been obtained by arranging the various observed radiants in related groups. In cases where single

radiants revealed a large number of meteors on one night these radiants have been included in spite of the lack of further corroboration on other dates.

The material for this paper was obtained mainly from the three reports of the New Zealand Meteor Section, covering the years 1927 to 1934, but recourse was also had to *Denning's General Catalogue of Meteor Radiants*, the publications of the American Meteor Society, the journals and memoirs of the British Astronomical Association, and Russian observations published by the Tashkent Observatory and Mirovédénié (Russian amateurs). These publications yielded a number of meteor radiants in southern declination. Although no claim is made that the index is complete, it is believed that little published material has been overlooked.

The task of grouping the radiants in their related families was a difficult one, in which a number of factors had to be kept steadily in view. The modern conception of radiation was the principal guide, and daily motion in radiants was looked for rather than stationary groupings. The effect of zenith attraction, placing northern observations too far north, had also to be allowed for.

It is only in recent years that a strict definition of meteoric radiation has been generally adopted. The writer was therefore faced with considerable difficulty in drawing a line of demarcation between good and bad in early lists of radiants. Where a rigid definition of radiation would have resulted in practically all the British radiants being rejected, a little elasticity justified the inclusion of radiants based on observations spread over several days.

Radiants covering intervals of several weeks were sometimes included when their positions were confirmed by modern and properly determined radiants, but the period of activity has been confined to that revealed by the modern observations. If this had not been done, bridges would have been provided linking distinct showers and providing apparently stationary radiants enduring for long periods.

For facility of reference the radiants have been arranged in order of right ascension. In the case of moving radiants they have been inserted at their earliest observed position, and their last observed position is also given. Where two or more radiants possess the same right ascension the northernmost have been given priority.

In pursuance of long-established practice each radiant has been given the name of the star which lies closest to it. Where no suitable star was available the constellation name only has been given. When several showers occur near the same star they have all been given the same name, but Roman numerals denote the order of their visibility.

An examination of the index reveals a large number of radiants which apparently are in daily motion, no less than 58 cases being given, in the majority of which the motion indicated seems probable. Now that attention has been drawn to them it is hoped that observers will give special attention to these showers so that the reality and extent of the motion can be established. Other radiants of which further observations would be

welcomed are the four cases in which stationary radiation is apparently established, namely, groups 114, 203, 264 and 283, the radiants in all cases lying close to the ecliptic. When additional data are available it will probably be found that these apparently stationary radiants will be resolved into a number of minor streams all showing the motion required by theory.

The New Zealand work reveals a great preponderance of observations in the months April to August, while the remaining months have not been satisfactorily covered. It would be inadvisable, therefore, to attempt at present to draw any conclusions as to the distribution of radiants.

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
1	July 26–Aug. 4	1 9	−15 −15	6	β Cet iv
2	Sept. 14–17	2	−16	2	6 Cet
3	Sept. 1–8	3	− 2	3	— Cet
4	July 26–28	5½	− 2½	2	— Cet
5	Oct. 30–Nov. 2	6	−15½	2	— Cet
6	July 15–26	8½ 17½	−22 −20½	4	β Cet ii
7	July 26	9	−20½	2	β Cet iii
8	Aug. 13–14	11	−31½	2	α Scl
9	Sept. 27	12	− 2	1	20 Cet
10	July 28–31	13½	−34½	2	σ Scl
11	July 26–30	15	−11½	4	η Cet
12	Aug. 8	15	−22	1	— Cet
13	July 26–27	15	−42	2	υ Phe
14	Aug. 5–9	16	− 4	2	34 Cet
15	Sept. 5–6	18	− 6	3	θ Cet i
16	Jan. 22–28	18	−20	2	— Cet
17	Aug. 16–20	19	−18	2	— Cet
18	July 19–20	21	−43	2	γ Phe
19	Aug. 1	21½	−11½	1	θ Cet ii
20	July 29–Aug. 15	25	−10	9	χ Cet
21	July 16–19	29	−20	2	υ Cet
22	July 26–31	30 37	−34 −37	2	— Phe
23	July 26–Aug. 4	31 39	−17 −16	5 5	— Cet
24	Oct. 25	31½	− 9	2	— Cet
25	Aug. 7–12	34	− 2	2	70 Cet
26	Oct. 18–19	34½	− 5½	2	67 Cet
27	Oct. 10–19	35	−12	3	ρ Cet
28	Oct. 25–Nov. 3	35½	−10½	4	— Cet
29	Nov. 12–19	37	−33	2	θ For
30	July 22–Aug. 1	38	− 2	3	75 Cet
31	Aug. 5	40	−44	1	— Eri
32	Oct. 5–10	41 47	− 3 − 6	2	δ Cet
33	Dec. 8–21	43	−60	2	— Hor
34	July 29–Aug. 4	45 49	−21 −21	4	— Eri

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
35	Oct. 15–23	47½	− 3	6	94 Cet
36	July 22	51½	−32½	1	— For
37	Jan. 4–12	56	−13	3	π Eri
38	Nov. 6–9	57	− 8	2	δ Eri i
39	Nov. 25–28	57	− 8	2	δ Eri ii
40	Oct. 5–6	59	−13	2	γ Eri
41	July 31–Aug. 1	65	−42	2	α Hor
42	Feb. 25–27	66	−63	2	— Ret
43	Sept. 6–13	68	− 3½	2	υ Eri
44	Oct. 30–31	72	− 0½	2	— Ori
45	Mar. 13–14	72	−53½	1	λ Pic
46	Oct. 20–21	73	−14	2	64 Eri
47	Sept. 29–30	75	−18	2	μ Lep
48	Nov. 27–Dec. 6	80	−30	3	— Col
49	Oct. 15–18	83	−14½	3	ζ Lep
50	Oct. 13–23	84	− 2	7	ζ Ori
51	Oct. 12–22	91 99	−14 −17	8	α CMa i
52	Oct. 18–21	92	− 4½	3	— Ori
53	Sept. 18–19	93½	− 0½	2	— Ori
54	Mar. 31	101	−16	1	α CMa ii
55	Dec. 3–6	101	−46½	2	X Pup
56	Nov. 10–19	103	−12	2	θ CMa
57	Feb. 11–16	103	−25	1	— CMa
58	Oct. 11–26	104	− 3	4	— Mon
59	Oct. 20–24	104	−13½	2	μ CMa
60	Oct. 24–30	112½	− 5	2	25 Mon
61	Dec. 4–12	113 117	−45 −47½	3	σ Pup
62	Jan. 10–15	114 120	−30 −29	2	p Pup
63	Nov. 15–16	114½	− 9	2	γ Mon
64	Mar. 16	116	− 8	1	— Mon
65	Dec. 3–4	117½	−40½	2	α Pup
66	Jan. 14–18	120	− 5	2	— Mon
67	Oct. 24–31	120½	− 6½	2	— Mon
68	Nov. 6–16	121 132	− 2 − 3	7	— Hya
69	Jan. 16–21	132	−54	3	δ Vel
70	Apr. 29–May 3	146	0	2	— Sex
71	Mar. 13–23	147	− 5	3	6 Sex
72	Jan. 24	149½	−24½	1	— Ant
73	Jan. 10	155	−25	1	— Ant
74	Apr. 15–28	156	−66½	2	θ Car
75	Jan. 1–11	157½	− 8	2	— Sex
76	Apr. 11–18	162½	− 1½	2	— Sex
77	Apr. 11–14	166	−16	2	δ Cra
78	Jan. 6	168	−59	1	χ Car
79	Apr. 9–10	170	− 1½	2	— Leo
80	Jan. 19	170	−63	1	— Car
81	Apr. 29–May 3	174	− 1	2	υ Leo
82	Mar. 7–14	174	−42	3	— Cen

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
83	May 9–11	176	−18	2	ζ Cra
84	Mar. 5–17	176	−56	2	— Cen
85	Jan. 16–20	177	−17	3	η Cra
86	June 28	178	− 5	1	— Vir
87	May 31	178	− 7	1	— Vir
88	Apr. 29–May 3	179	−10	2	— Vir
89	Apr. 12	180	−17	1	— Crv
90	Apr. 29–May 11	180 186	−21 −20	4	ϵ Crv
91	Apr. 12–16	184 188	0 0	2	η Vir
92	Jan. 12	184	−64	1	α Cru
93	May 9	185	−13	1	— Crv
94	May 31	185	−20½	2	ζ Crv
95	Mar. 18–25	186½	−18	2	δ Crv
96	Apr. 9–19	186 195	− 8 − 7	7	χ Vir
97	Apr. 16–21	188½	−31½	2	— Hya
98	May 1–12	189	− 5	2	γ Vir
99	May 11	191	−58	2	β Cru
100	Feb. 18–20	191½	−33	2	— Cen
101	Apr. 13–14	192	−14½	2	— Vir
102	Mar. 19–31	194	− 3	2	46 Vir
103	Apr. 29–May 12	196	− 6	6	θ Vir
104	Apr. 10	196	−34	1	ι Cen
105	Apr. 8–11	196	−45½	2	— Cen
106	Apr. 30–May 9	198 203	−22 −19	3	γ Hya ii
107	Jan. 24–Feb. 3	198	−24	3	γ Hya i
108	May 31	199	−10	1	α Vir ii
109	Apr. 9–21	202 214	−30 −30½	4	4 Cen
110	Mar. 7–11	203	−34	3	T Cen
111	Jan. 10–14	204	− 6½	2	74 Vir
112	Feb. 10–13	206	−43	2	μ Cen
113	Apr. 12–14	208	− 2½	5	— Vir
114	Apr. 4–30	208½	−10	24	α Vir i
115	May 3–11	211	−41	2	χ Cen
116	Apr. 9–11	212 216	−24 −26	3	π Hya
117	Feb. 15–21	214	−13	2	λ Vir
118	Apr. 29–May 3	214	−41	3	η Cen
119	Apr. 19–23	215	− 6	4	ι Vir
120	May 11	215	−10	1	κ Vir
121	Mar. 20–28	217	−45	2	τ Lup
122	Mar. 26–28	218	−13	2	α Lib
123	Apr. 21–24	218½	−27	3	51 Hya
124	July 7	220	−25	1	4 Lib
125	Apr. 10	220	−48	1	α Lup
126	Apr. 10–14	220	−65	2	α Cir
127	Apr. 9–26	225	− 3	7	— Lib
128	Apr. 17–21	226	−22	6	ι^1 Lib

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
129	June 25	227	− 5	1	— Lib
130	Apr. 4–6	230	−21	1	— Lib
131	Apr. 19–22	231	− 8	3	β Lib
132	May 9–11	232	−15	2	γ Lib
133	May 3–9	233	−23½	3	42 Lib
134	Apr. 13–21	233½ 238½	−19 −22	9	κ Lib
135	May 18–23	234	− 2	2	μ Ser ii
136	June 6–12	234	−21½	2	λ Lib i
137	Mar. 2–9	235	−17	4	η Lib
138	Apr. 1–13	235½	−18½	3	θ Lib
139	May 31	236	− 2	1	μ Ser iii
140	Mar. 27	236	− 3	2	μ Ser i
141	Apr. 17–19	236	−15	3	48 Lib
142	July 8–11	237	− 3	3	— Ser
143	June 28–July 7	237 251	−18 −18	13	λ Lib ii
144	July 5–8	238	−19	9	β Scr
145	June 8–13	239 248	0 − 1	5	χ Ser
146	May 29–June 11	240 247	−21 −18	8	ω^2 Scr
147	May 28–June 11	241 247	−13 −18	5	χ Scr
148	Apr. 9–11	241	−25½	2	σ Scr
149	Mar. 6–11	241	−53	3	— Scr
150	Apr. 19–May 9	242 254	−25 −23	6	ρ Oph
151	July 7–8	242	−39	3	— Lup
152	May 31	243	−27½	2	13 Scr
153	May 11–16	243	−32	2	— Scr
154	May 10–11	243	−48	2	γ^2 Nor
155	July 7	244	−39	1	— Nor
156	July 5–8	245 248	− 7 − 9	5	v Oph
157	May 6–15	246	−26	4	α Scr
158	Apr. 4–17	247½ 253	−50 −53	4	ρ^2 Ara i
159	May 17–24	248	−22½	2	ω Oph
160	May 7–14	248½	−15	3	ϕ Oph
161	May 15	249½	−25	1	25 Scr
162	June 1–15	250	−10	3	ζ Oph
163	June 1–10	250 259	−22 −23	9	ξ Oph ii
164	July 8	252	−30	1	— Scr
165	June 6–15	254	−11½	4	— Oph
166	June 15	254	−51	1	ρ^2 Ara ii
167	May 31	256	−17	2	η Oph
168	Apr. 27–May 5	257	− 1	2	— Oph
169	July 5–9	257 261	− 8 −12	9	μ Oph
170	May 19–22	258	−23	3	ξ Oph i

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
171	Apr. 13–21	260	−36	3	υ Scr
		266	−36		
172	Apr. 3–6	261½	−32	1	— Oph
173	June 30–July 10	262	−20	10	μ Sgr
		275	−20		
174	July 9	263	−29½	2	45 Oph
175	July 1–9	263½	−21½	2	52 Oph
176	July 21–28	264	−18	3	ξ Ser
		269	−20		
177	June 7	264½	−42½	1	θ Scr
178	June 8–9	265	−13½	4	o Ser i
179	June 21–July 2	265	−21	4	58 Oph
180	June 14–21	266	−18½	2	— Oph
181	July 20	266	−33	2	— Sgr
182	Aug. 10–11	267	−12	2	o Ser iii
183	Aug. 1–6	267	−12½	2	o Ser ii
184	June 14–15	268	−26	3	3 Sgr
185	Apr. 13–22	269	−10	7	ν Oph
186	May 2–11	270½	−27	8	γ^1 Sgr
187	Aug. 4–13	271	− 3	2	η Ser ii
188	Mar. 13–28	272	−40	5	λ CrA
		283½	−38		
189	June 8–10	273	−13	3	— Aql
190	July 10–17	273	−36	2	ϵ Sgr ii
		277½	−37		
191	June 25–30	274	− 6	3	η Ser i
		277	− 3		
192	Oct. 22	275	−15½	1	— Aql
193	Apr. 10–22	275	−32	8	δ Sgr
		278	−34		
194	June 8–11	276	− 1½	4	61 Ser
195	June 8–15	276	−28	8	ϕ Sgr
		285	−28		
196	May 4–5	276½	−25	4	λ Sgr ii
197	June 14–19	276½	−26	3	λ Sgr iii
198	June 8–14	277	−35	2	ϵ Sgr i
199	July 30–Aug. 4	278	− 6	2	6 Aql
200	June 15	278	− 8	1	1 Aql
201	June 10–19	280	−11	8	— Aql
		291	−12		
202	July 30–Aug. 5	282	−13	2	— Sgr
203	July 2–19	283	−15	9	— Sgr
204	June 16	284	− 3½	1	λ Aql
205	May 4–7	284	−17	3	— Sgr
206	Aug. 6–10	284	−20½	2	ξ^1 Sgr
207	June 1	284	−36	2	ϵ CrA ii
208	Apr. 28–May 5	284	−37	4	ϵ CrA i
209	May 12	284½	−34	1	— Sgr
210	July 10–20	285	−25	4	ψ Sgr iii
211	June 28–July 5	285	−25	6	ψ Sgr ii
212	Apr. 27–May 3	285	−26	3	ψ Sgr i
213	Apr. 4–6	285	−53	1	ρ Tel i

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
214	July 28–Aug. 4	286	−37	3	γ CrA
215	May 3–4	286	−53	2	ρ Tel ii
216	Mar. 22	286	−57	1	μ Tel
217	July 25–31	288	− 8	2	20 Aql
218	June 15–22	290	−18	4	ρ^1 Sgr
219	June 15–22	291 297	−27 −25	2	χ^1 Sgr
220	July 23–Aug. 4	292 297	−10 −11	3	37 Aql ii
221	Aug. 1–7	292 297	−23 −22	4	50 Sgr i
222	July 5–7	292½	− 8½	2	κ Aql
223	June 6–8	293	−13½	2	37 Aql i
224	Dec. 8–17	293	−21	2	50 Sgr ii
225	Aug. 4–6	294	0	3	ι Aql
226	July 23–30	294½	−10½	2	37 Aql iii
227	July 14–23	295	−23	3	53 Sgr
228	Sept. 20–24	295	−32	2	— Sgr
229	May 2–4	295	−42	2	ι Sgr
230	July 15–16	295½	−13	2	55 Sgr
231	July 29–Aug. 1	296	− 6	3	— Aql
232	July 13–24	297	0	4	η Aql
233	May 3–7	298	− 3½	4	— Aql
234	July 23–31	300 305	− 9 − 8	6	— Cap
235	July 22–31	300 308	−11 −10	15	α Cap ii
236	May 4–11	301	−24	3	— Sgr
237	June 28–July 5	302	− 5½	5	— Aql
238	July 4–7	302	− 9	3	α Cap i
239	July 10–20	302 317	−14 −11	10	β Cap ii
240	July 10–15	302 313	−17 −19	12	σ Cap
241	May 3–11	303	−14	5	β Cap i
242	July 28–Aug. 6	303	−47	3	α Ind ii
243	July 10–18	303½	−13½	5	ξ Cap
244	Apr. 13–15	304	−66½	2	δ Pav
245	Apr. 16–23	304½	−51	2	η Ind
246	May 1–6	307½	−17	2	ρ Cap
247	May 7–8	308	−49	3	α Ind i
248	July 2–6	311	−14	3	τ^2 Cap
249	July 16–23	312	−43½	2	ι Mic
250	July 18–29	313	−30	4	— Cap
251	Aug. 8–15	314	−10	7	7 Aqr
252	May 2–6	314	−34	2	ν Mic i
253	July 4–10	314	−35	2	ν Mic ii
254	July 30	317	− 9	2	— Aqr
255	Aug. 9–12	320	−15	3	ι Cap
256	Nov. 25	321	− 5	1	β Aqr ii
257	July 31–Aug. 8	321	− 5½	6	β Aqr i
258	June 1	322	−53½	1	θ Ind

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
259	July 26–30	323	−41	3	θ^2 Mic
260	Apr. 29–May 7	323½	−17	9	γ Cap
261	July 8–14	324	−53½	2	δ Ind
262	July 21–Aug. 12	324½	− 4	8	— Aqr
263	July 2–5	325	− 4	2	— Aqr
264	July 18–Aug. 5	325	−15	26	δ Cap
265	May 4–8	325	−34	8	ι PsA
266	July 28–Aug. 3	326	−26	3	— PsA
267	July 10–11	327	0	3	— Aqr
268	Apr. 30–May 8	327	− 2	6	o Aqr
269	July 2–4	327	−24½	2	— PsA
270	Aug. 12–14	327	−32	2	θ PsA
271	Nov. 3–6	328½	−12	2	μ Cap
272	July 31–Aug. 11	330 339	−14 −10	7	ι Aqr ii
273	Apr. 28–May 16	330½ 346½	− 3 + 3½	27	η Aqr. Maximum May 5
274	July 14–22	330½ 339	−30 −30	11	β PsA ii
275	July 28–Aug. 2	331 334	− 8 − 8	11	θ Aqr
276	July 1–12	331 339	−13 −12	4	τ^1 Aqr
277	Aug. 12–23	331 339	−13 −10	6	τ^2 Aqr
278	June 24–July 1	331 337	−47 −48	2	α Gru
279	Aug. 3–8	332	− 1	2	γ Aqr
280	July 25–Aug. 5	332 338	−15 −12	13	ι Aqr i
281	Aug. 5–14	334 339	−27½ −26	5	λ PsA
282	July 2–4	334	−31	3	— PsA
283	July 25–Aug. 15	335	0	23	ζ Aqr i
284	Sept. 1–14	335	− 2	3	ζ Aqr ii
285	July 22–Aug. 9	335 351½	−17 −12	44	δ Aqr. Maximum July 28
286	May 4–5	336	−28	1	ζ PsA
287	May 12	336	−33½	2	β PsA i
288	Sept. 7–8	337	−11	2	σ Aqr
289	July 28–Aug. 3	337	−13	3	70 Aqr
290	July 26–Aug. 8	337 350	−33 −30	24	α PsA
291	Aug. 13–14	338	−24	3	ϵ PsA
292	July 20–Aug. 5	340 346	− 8 − 7	3	λ Aqr i
293	Aug. 8–9	340½	−27	2	20 PsA
294	Aug. 10–20	341 347	−12 −12	5	74 Aqr ii
295	Aug. 13–14	342	− 7	3	λ Aqr ii
296	July 28–Aug. 1	342	−12	3	74 Aqr i
297	Aug. 6–7	343	−17½	2	77 Aqr

No.	Period	Radiant 1900 α	Radiant 1900 δ	No. Rads.	Name
		°	°		
298	June 14–22	344	−45	2	θ Gru ii
299	Sept. 14–23	345	0	6	3 Psc
300	Aug. 25	345	−45	1	ι Gru
301	Aug. 13–21	346	0	4	5 Psc i
302	Aug. 24–Sept. 9	346	− 0½	2	5 Psc ii
303	May 4–5	346	−46	2	θ Gru i
304	Aug. 16–20	346½	−27½	2	— PsA
305	July 9–12	347	0	2	— Psc
306	July 15–31	347	− 1	5	— Psc
307	July 28–Aug. 1	347	−58	2	γ Tuc
308	July 2–8	348½	− 9	5	ψ^2 Aqr
309	July 15–31	349	− 1	4	— Psc
310	July 28–Aug. 1	349	−32	2	μ Scl
311	Aug. 6–10	350 359	−18 −14	5	98 Aqr
312	Aug. 7–13	351	−21	4	101 Aqr
313	July 20–26	351	−38½	2	— Scl
314	Aug. 13–16	351	−62½	2	η Tuc
315	Aug. 13–25	352	− 3	4	14 Psc
316	July 19–Aug. 1	352 11	−18 −18	13	β Cet i
317	Oct. 16–21	352½	− 6	2	24 Psc
318	July 19–30	354	−15	5	ω^2 Aqr
319	Aug. 1–11	354 4	−20½ −20	6	108 Aqr
320	July 28–Aug. 2	357	− 5	5	27 Psc